MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

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Professor H. C. Plummer, President, in the Chair

The election by the Council of the following Fellows was duly confirmed :—

Dr. A. E. Benfield, Thompson Physical Laboratory, Williams College, Williamstown, Mass., U.S.A. ;
Douglas Charles Berry, Lands and Survey Department, Invercargill, New Zealand ;
William Edward Browne, F.S.I., F.R.G.S., c/o Anglo-Iranian Oil Co. Ltd., Britannic House, Finsbury Circus, London, E.C. 4 ;
Hugh Ernest Butler, B.A., Emmanuel College, Cambridge ;
Edward Hope-Jones, B.A., c/o H.M. Office of Works, Contract No. 4, R.O.F., Glascoed, Usk, Mon. ;
David G. Kendall, 9 Clotherholme Road, Ripon, Yorks ;
Thomas James Latus, 7 Willoughby Road, Hampstead, N.W. 3 ;
Edward Hubert Linfoot, D.Phil., 13 Alexander Road, Clifton, Bristol 8 ;
John Felix de Lisle, M.Sc., 16 View Road, Mount Eden, Auckland, S. 1, New Zealand ;
Rev. Charles Linaker Lomas, The Rectory, Badlesmere, near Faversham, Kent ;
Ernest Henry Noon, 23 Belltrees Grove, Streatham, London, S.W. 16 ;
Roger W. Pring, B.Sc., 34 Village Road, Finchley ;
J. J. Shaw, C.B.E., M.Sc., Sunnyside, West Bromwich ;
Arthur Albert Smith, B.Sc., Sevenoaks School, Kent ;
Dr. Lyman Spitzer, Jr., Sloane Physics Laboratory, Yale University, New Haven, Conn., U.S.A. ;

Leslie Granville Taylor, 17 Cawdor Crescent, Boston Manor, London, W. 7 ; and

Eric Bell Walton, A.M.I.C.E., A.R.I.B.A., 42 Hainault Avenue, Westcliff, Essex.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended :—

E. Finlay Freundlich, Ph.D., Lecturer in Astronomy, The Observatory, University of St. Andrews (proposed by W. M. H. Greaves).

Fifty-four presents were announced as having been received since the last meeting.

The President announced that the Gold Medal of the Society had been awarded by the Council to Dr. Edwin P. Hubble, the grounds of the award being his outstanding work on the distances, velocities, distribution and nature of the extra-galactic nebulæ.

THE VARIATION OF LATITUDE

Harold Jeffreys, F.R.S.

There appears to have been no systematic attempt to estimate the components of the variation of latitude since that of L. W. Pollak * ; and meanwhile many more years' observations have accumulated. The best procedure will depend to some extent on the physical explanation of the phenomena, which are generally recognized to consist of a free motion of the Earth's axis of rotation about a principal axis, with a period of about 14 months (Pollak considered 1·20 years better) and a superposed annual motion in an ellipse, usually believed to be due to meteorological changes, which produce periodic changes in the Earth's products of inertia about the mean position of the principal axis. Most attempts to determine the constants have begun with the 14-monthly period, presumably because it is associated with the larger amplitude ; but this procedure is open to some objections. Undamped free vibrations in the sense of elementary dynamics probably have no physical existence ; when we observe a maintained free vibration there is always some supply of new energy to replace what is dissipated. The motion of a piano string after it is struck, for instance, is a damped free vibration. The motion of a violin string is only semi-periodic ; the apparent steady amplitude is maintained by irregularities in the friction of the bow and is associated with continual changes of phase. Similarly, the energy of the sound from an organ-pipe is maintained by the unstable jet at the mouthpiece ; it is the tendency of the displacements of this jet to grow that contributes the new energy to the pipe. It seems therefore that analysis of an apparent free vibration as if it was strictly periodic is hazardous ; its maintenance in spite of dissipation involves forces out of phase with the displacement, and these are not necessarily even periodic. A closer analogy with the problem of the variation of latitude, and probably with most free vibrations as they occur in nature, is one suggested by G. Udny Yule.† A heavy pendulum is bombarded by a number of boys with pea-shooters, whose rate of firing and aim are both imperfect. Consequently the pendulum receives a series of impulses at irregular intervals and a vibration is produced. But this is not strictly periodic, and if we adopt any given period there will be considerable variations of amplitude and phase from time to time. Harmonic analysis over a long interval may fail to reveal a definite periodicity at all, because if irregular disturbances started the pendulum in one phase, new ones may very well reverse it later. Nevertheless the pendulum has a natural period, with some damping, and it is legitimate to ask how this can be found from observations of its displacement at regular intervals ; particularly when there is no record of when the peas actually did hit it. Yule shows that suitable information is provided by the correlation between the

* *Gerlands Beitr.*, **16**, 108–194, 1927.
† *Phil. Trans.*, A, **226**, 267–298, 1927.

displacements at different intervals apart. It can be seen that even though the motion may be irregular and possibly undergo great changes of phase in a long time, the correlation between displacements at times a quarter-period apart will be practically zero, and the period can be determined from examination of the intervals such that the correlation vanishes.

The distribution of air over the Earth's surface, though mainly periodic, is not strictly so ; it does not accurately repeat itself every year. It is therefore possible that the free variation of latitude is maintained by the irregular variation of the products of inertia, superposed on the mean annual variation. It is also implied by dynamics that the motion will be associated with a small oceanic tide, which, combined with the tidal motion, will be subject to dissipation. Hence, except for the complication of gyroscopic coupling between the two components of the displacement, the problem is similar to that of the bombarded pendulum.

This suggests that the best treatment will involve determining and eliminating the mean annual movement first. This has an exact known period, and, if we determine it from sets of equal intervals, comparison of the separate values will provide an estimate of accuracy. It will be best to choose the intervals, as far as possible, to be multiples of the free period ; then the free motion will contribute little to the harmonic terms if it is regular, and if it is irregular its contributions will be random and will be included in the estimate of uncertainty. If we adopt a trial interval that is not a multiple of the free period, the free motion will contribute an additional part to the estimates of the harmonic terms. This will not always have the same sign and will therefore increase the variance among the intervals. The best interval, so far as the elimination of the free motion is concerned, will be the one that gives the smallest variation of the coefficients of the annual terms.

The data were taken from various publications of the International Variation of Latitude Service mainly by Kimura, except for 1912 to 1923, the data for which are given by Wanach. In some cases the data have been readjusted ; the latest was taken in each such case. In the tabulations x is the displacement of the north pole of the instantaneous axis towards Greenwich and y the displacement to 90° W. The data are at intervals of 0·1 year and the unit is 0″·01. In the first place a form $a \cos L + b \sin L$ was found for the sums at corresponding L over ranges of 6 years, and gave the following determinations. L is the Sun's longitude from the beginning of the year.

	x		y	
	a	b	a	b
1892–97	−18	−66	−30	+ 7
1898–1903	−15	−45	−32	+ 4
1904–1909	−27	−28	−21	+16
1910–15	−17	−53	−61	+18
1916–21	−25	−51	−48	+33
1922–27	−10	−56	−53	+ 1
1928–33	−39	−57	−49	+22
Mean	−22	−51	−42	+14

For seven-year ranges the determinations were as follows :—

	x		y	
	a	*b*	*a*	*b*
1892–98	− 9	−89	−42	− 2
1899–1905	−17	−51	−46	− 2
1906–12	−22	−55	−55	+ 6
1913–19	−30	−38	−49	+49
1920–26	−34	−65	−53	+25
1927–33	−41	−59	−51	+24
Mean	−25	−60	−49	+17

The sums of the squares of the residuals are

	x		y		Total
	a	*b*	*a*	*b*	
Six-year ranges	557	855	1252	763	3427
Seven-year ranges	691	1456	114	1980	4241

The expectations of these totals would be the same if the variation was random ; but since they would be inflated by imperfect elimination of the free motion it is probable that the first is the better. Then the free period is probably nearer to 6/5 than to 7/6 years, in agreement so far with Pollak. The sum 3427 on 24 degrees of freedom gives 12·0 for the standard error of a harmonic coefficient in a six-year sum, $12{\cdot}0/\sqrt{7}$ for the mean of seven and $12{\cdot}0/6\sqrt{7}=0{\cdot}76$ for the average value for a single year. The sums over the whole 42 years were as follows :—

	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
x	− 86	−265	−334	−249	− 48	+215	+397	+447	+329	+123
y	−239	−106	+ 77	+258	+346	+349	+254	+ 72	−130	−236

Direct harmonic analysis and reduction to one year gave the following :—

$$x = -3{\cdot}62\cos L - 8{\cdot}49\sin L,$$
$$y = -6{\cdot}99\cos L + 2{\cdot}93\sin L,$$

the standard error of each coefficient being 0·76. It seemed worth while to examine the totals also for periods of $\frac{1}{2}$ and $\frac{1}{3}$ year, but no coefficient reached 0·2, which is less than the standard error, and it appears that periodicities in submultiples of a year are not significant. The annual terms, in units of 0″·01, at intervals of 0·1 year, are :

	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
x	−4	−8	−9	−7	−2	+4	+8	+9	+7	+2
y	−7	−4	+1	+5	+7	+7	+4	−1	−5	−7

At this stage it was noticed that the sums of the values of x and y are positive, and would need constant terms to be added to express them. Presumably they are due to errors in the adopted mean latitudes of the observatories. They would not affect the harmonic analysis, but they might affect the further work because a constant addition to x, say, would contribute a positive

part to the correlation between the values of x at different times and perhaps lead to a systematic error in the free period. On examining the 6-year intervals separately the sums were found to differ considerably; they are as follows :—

	x	y
1892–97	+ 5	+ 16
1898–1903	+ 49	− 79
1904–09	+ 6	0
1910–15	+206	+204
1922–27	+100	+ 92
1928–33	−146	+217
1934·0–1938·1	−195	+221

Each of these is the sum of 60 individual entries except the last pair, which represent 41 each, and it appears that they may affect the entries by over 0″·03, a considerable fraction of the amplitude of the annual motion. They are clearly not explicable by the inclusion of an occasional extra contribution from the free terms. Only a very rough interpolation is possible, but yearly values in multiples of 0″·01 can be taken to be as follows :—

	x	y		x	y		x	y		x	y
1891	0	+1	1903	+1	−1	1915	+4	+4	1927	0	+2
1892	0	+1	1904	0	−1	1916	+4	+3	1928	0	+2
1893	0	+1	1905	0	0	1917	+5	+3	1929	−1	+3
1894	0	+1	1906	0	0	1918	+5	+3	1930	−2	+3
1895	0	0	1907	0	0	1919	+5	+3	1931	−3	+4
1896	0	0	1908	0	0	1920	+5	+3	1932	−3	+4
1897	0	0	1909	+1	+1	1921	+4	+3	1933	−4	+4
1898	0	−1	1910	+2	+2	1922	+4	+2	1934	−5	+5
1899	+1	−1	1911	+2	+2	1923	+3	+2	1935	−5	+5
1900	+1	−1	1912	+3	+3	1924	+2	+2	1936	−5	+5
1901	+1	−1	1913	+4	+4	1925	+2	+1	1937	−5	+5
1902	+1	−1	1914	+4	+4	1926	+1	+2	1938	−5	+5

If l, m are direction cosines of the instantaneous axis, with respect to right-handed axes fixed in the Earth, the equations of motion, including a damping term, may be written

$$\dot{l}+kl+\gamma m=0\,; \qquad \dot{m}+km-\gamma l=0. \tag{1}$$

If the initial values of l and m are l_0, m_0, we write the subsidiary equations

$$(p+k)l+\gamma m=pl_0\,; \qquad (p+k)m-\gamma l=pm_0, \tag{2}$$

and the operational solution is

$$l=\frac{p(p+k)l_0-\gamma pm_0}{(p+k)^2+\gamma^2}\,; \qquad m=\frac{p(p+k)m_0+\gamma pl_0}{(p+k)^2+\gamma^2}, \tag{3}$$

which gives on interpretation,

$$\left.\begin{aligned} l&=l_0e^{-kt}\cos\gamma t-m_0e^{-kt}\sin\gamma t,\\ m&=l_0e^{-kt}\sin\gamma t+m_0e^{-kt}\cos\gamma t.\end{aligned}\right\} \tag{4}$$

If the observations are at intervals of time τ, and there is a disturbance ϵ_n, η_n in any interval, we shall have the recurrence relations

$$l_n = \alpha l_{n-1} - \beta m_{n-1} + \epsilon_n, \tag{5}$$

$$m_n = \beta l_{n-1} + \alpha m_{n-1} + \eta_n, \tag{6}$$

where

$$\alpha = e^{-k\tau} \cos \gamma\tau\,; \qquad \beta = e^{-k\tau} \sin \gamma\tau. \tag{7}$$

Now suppose that the disturbances are independent, ϵ_n and η_n each having standard error σ. Then the joint probability of pairs of values of l and m will follow the rule:

$$P(dl_1 dl_2 \ldots dl_n dm_1 \ldots dm_n \mid l_0 m_0 \alpha\beta H) = \frac{1}{(2\pi\sigma^2)^n} \exp\left[-\frac{1}{2\sigma^2} \sum_{r=1}^{n} \{(l_r - \alpha l_{r-1} + \beta m_{r-1})^2 + (m_r - \beta l_{r-1} - \alpha m_{r-1})^2\} \right] dl_1 \ldots dl_n dm_1 \ldots dm_n \tag{8}$$

$$= \frac{1}{(2\pi\sigma^2)^n} \exp\left[-\frac{1}{2\sigma^2} \{\Sigma l_r^2 + \Sigma m_r^2 + (\alpha^2+\beta^2)(\Sigma l_{r-1}^2 + \Sigma m_{r-1}^2) - 2\alpha\Sigma(l_r l_{r-1} + m_r m_{r-1}) + 2\beta\Sigma(l_r m_{r-1} - m_r l_{r-1})\} \right] dl_1 \ldots dm_n. \tag{9}$$

It appears from (8) that it would be possible to use (5) and (6) as a set of equations of condition to determine α and β if the l_r and m_r were observed. This is analogous to Yule's procedure. But there is a strong possibility of bias, since the observed values have observational errors and are not the l and m of the above equations. We can see this by considering correlations. The correlation between l_n and l_{n+p} is, by (4), equal to $\exp(-kp\tau) \cos(\gamma p\tau)$, since l_n is not correlated with disturbances at later times, and the only part of l_{n+p} correlated with l_0 is the residual effect of l_0 itself. Then similarly

$$\left.\begin{aligned} \rho(l_n, m_{n+p}) &= \exp(-kp\tau) \sin \gamma p\tau, \\ \rho(m_n, l_{n+p}) &= -\exp(-kp\tau) \sin \gamma p\tau, \\ \rho(m_n, m_{n+p}) &= \exp(-kp\tau) \cos \gamma p\tau. \end{aligned}\right\} \tag{10}$$

The normal equations found from (5) and (6) will be

$$\left.\begin{aligned} \alpha\Sigma(l_{n-1}^2 + m_{n-1}^2) &= \Sigma(l_n l_{n-1} + m_n m_{n-1}), \\ \beta\Sigma(l_{n-1}^2 + m_{n-1}^2) &= \Sigma(l_{n-1} m_n - l_n m_{n-1}), \end{aligned}\right\} \tag{11}$$

and the correlations between consecutive values of l and m contain all the information needed to determine α and β. The effect of observational error, however, is to increase the sums on the left while not systematically affecting those on the right. (This assumes that the observational errors at intervals of 0·1 year are independent; this may not be strictly true, but at least observational error will increase the left sides more than the right.) It will therefore lead to underestimating α and β in the same ratio, or, having regard to their forms, to an overestimate of the damping while leaving the period unaffected. Similar considerations affect Yule's method applied to a single component, which Dyson attempted to use on the Greenwich observations of the variation of latitude but finally rejected because it led to

an impossible estimate of the damping.* If we correct for the error of observation, supposed to have standard error σ' in each component, the equations will become

$$\left.\begin{aligned}\alpha(\Sigma l_{n-1}^2+\Sigma m_{n-1}^2-2N\sigma'^2)&=\Sigma(l_n l_{n-1}+m_n m_{n-1}),\\ \beta(\Sigma l_{n-1}^2+\Sigma m_{n-1}^2-2N\sigma'^2)&=\Sigma(l_{n-1}m_n-l_n m_{n-1}),\end{aligned}\right\} \quad (12)$$

and we need a further equation to separate the damping from σ'. This can be provided by considering observations p intervals apart. They will lead to

$$e^{-2pk\tau}(\Sigma l_{n-p}^2+\Sigma m_{n-p}^2-2(N-p)\sigma'^2)^2=\{\Sigma(l_n l_{n-p}+m_n m_{n-p})\}^2 + \{\Sigma(l_{n-p}m_n-l_n m_{n-p})\}^2, \quad (13)$$

and comparison of this equation with that for $p=1$ will give k. A little examination shows that the best value of p to choose is such that the damping factor $\exp(-pk\tau)$ is about 0·45. Too small a value of p will lead to loss of accuracy because the logarithm of the ratio of the correlations has to be divided by $p-1$; too large a value because the correlations at wide intervals will be small and badly estimated in relation to their uncertainties.

A little preliminary treatment is worth while. We may take the means of successive (not overlapping) sets of three observed values as our data instead of the individual values. This loses little accuracy, because it is known that heavy grouping leads to little loss of efficiency in harmonic analysis, and this loss will be more than compensated by the increased probability that disturbances in different intervals of 0·3 year rather than 0·1 year will be independent. It also tends to reduce the importance of observational error in relation to the real variation. Overlapping means of course cannot be used because their uncertainties are not independent.

The axes of x and y in the publications are left-handed; the relation to l and m (observed values of the latter being understood) is

$$x=l; \qquad y=-m.$$

From the means at intervals 0·3 year we find:

$$\begin{array}{lll}\Sigma x_{n-1}^2 = 13287; & \Sigma y_{n-1}^2 = 14045; & \Sigma x_{n-1}^2+\Sigma y_{n-1}^2 = 27332;\\ \Sigma x_n x_{n-1} = +236; & \Sigma y_n y_{n-1} = +504; & \Sigma x_n x_{n-1}+\Sigma y_n y_{n-1} = +740;\\ \Sigma x_{n-1}y_n = -12229; & \Sigma x_n y_{n-1} = +12900; & \Sigma x_{n-1}y_n-\Sigma x_n y_{n-1} = -25129;\end{array}$$

and the normal equations are (the factor λ expressing the correction for observational error)

$$27332\lambda\alpha=+740,$$
$$27332\lambda\beta=+25129.$$

The sum of squares of 308 residuals is 4040; whence

$$\lambda\alpha=+0{\cdot}0271\pm0{\cdot}0219,$$
$$\lambda\beta=+0{\cdot}9194\pm0{\cdot}0219,$$
$$\sigma^2=13{\cdot}2; \qquad \sigma=3{\cdot}64;$$
$$\cot\gamma\tau=\alpha/\beta=+0{\cdot}0295\pm0{\cdot}0238,$$
$$\gamma\tau=88^\circ{\cdot}31\pm1^\circ{\cdot}38,$$

* *Bull. Calcutta Math. Soc.*, **20**, 23–30, 1928.

with $\tau = 0{\cdot}3$ year. Hence the period is $1{\cdot}223 \pm 0{\cdot}019$ year

$$= 446{\cdot}7 \pm 6{\cdot}9 \text{ mean solar days}$$
$$= 447{\cdot}9 \pm 7{\cdot}0 \text{ sidereal days.}$$

The free period is therefore probably a little over 1·2 years, or about 14·7 mean months. We have also

$$\lambda e^{-0{\cdot}3k} = \lambda(\alpha^2 + \beta^2)^{\frac{1}{2}} = 0{\cdot}9198 \pm 0{\cdot}0220.$$

If λ was 1, that is, if there were no error of observation, this would suggest a decay of amplitude in the ratio e to 1 in about 3 years. This suggested carrying out a similar analysis on observations separated by 2·4 years. This adds nothing to the accuracy of the period; the errors also are not independent because any disturbance occurring in a particular 0·3-year interval will contribute to 8 intervals of 2·4 years. Hence the uncertainties were multiplied by $\sqrt{8}$ to allow for this. A further analysis of intervals of 15·0 years was also done, the uncertainty this time being multiplied by $\sqrt{45}$. This gave the three equations for λ and k:

$$\lambda e^{-0{\cdot}3k} = 0{\cdot}92(1 \pm 0{\cdot}024),$$
$$\lambda e^{-2{\cdot}4k} = 0{\cdot}81(1 \pm 0{\cdot}12),$$
$$\lambda e^{-15{\cdot}0k} = 0{\cdot}393(1 \pm 0{\cdot}22),$$

and the least squares solution is

$$k = 0{\cdot}0577 \pm 0{\cdot}0145; \qquad \lambda = 0{\cdot}933(1 \pm 0{\cdot}023).$$

Then k suggests that the time of decay is 14 to 24 years. But these values imply that only about 0·3 of the departure of the first correlation from unity is due to damping; the rest is due to observational error. In that case the contribution of observational error to each residual corresponds to a standard error of about 3·5 units. This is not unreasonable, for consecutive values enter with coefficients near 1; the result would correspond to a standard error of $3{\cdot}5(3/2)^{\frac{1}{2}} = 4{\cdot}3$ units in one of the original values at intervals of 0·1 year. Kimura's estimate is 3 units. But it may well affect the basis of our estimates, since we have supposed that the disturbances contribute independently to the equations of condition and that observational error can be allowed for by a small correction. We find now that observational error contributes a large part of the differences between consecutive values, and a given observational error appears in three equations of condition, so that the errors of the equations are far from independent. The error of l_n, for instance, appears not only in the equation for l_n but in those for l_{n+1} and m_{n+1}.

An attempt to estimate the observational error more closely was made by applying harmonic analysis for a 1·2-year period directly to the observed values, at 0·1-year intervals, after correcting for the annual term and the above mean departures. Ranges of 1·2 years were analysed separately. The following table gives the results. The constant term found is denoted by c; accents denote values derived from y. The date given is that of the first entry used in the summary.

	c	a	b	c'	a'	b'	σ	σ'
1891·5	+2	+ 6	+19	−3	+11	− 6		8·0
1892·7	+2	+ 5	+12	−1	+13	− 3		
1893·9	+2	− 3	+10	−1	+10	− 5	3·9	2·56
1895·1	−2	+ 3	+ 4	−1	+12	−11		
1896·3	−1	+ 3	+10	0	+12	− 6		
1897·5	−1	0	+19	+2	+10	+ 2		
1898·7	+3	+ 6	+ 8	+3	+16	− 7		
1899·9	−2	+ 9	+11	−3	+14	− 6	3·1	1·91
1901·1	+1	+ 8	+11	−5	+10	− 9		
1902·3	+1	+10	+ 9	+2	+10	− 7		
1903·5	0	+ 6	+ 9	+2	+10	− 6		
1904·7	0	+ 8	+ 9	0	+13	− 9		
1905·9	−2	+14	+ 6	+2	+ 6	−13	2·05	1·89
1907·1	−1	+18	+ 3	+1	+ 8	−20		
1908·3	0	+24	+ 9	0	+ 5	−22		
1909·5	0	+24	+ 6	−2	+ 9	−22		
1910·7	+1	+20	+ 6	0	+ 5	−26		
1911·9	+1	+21	+ 8	0	+ 5	−21	2·36	1·91
1913·1	+1	+18	+ 8	−2	+ 8	−17		
1914·3	0	+18	+ 6	+2	+ 9	−22		
1915·5	−1	+15	+ 3	+4	+ 6	−17		
1916·7	0	+13	+ 6	0	− 2	−14		
1917·9	−2	+12	+ 2	+2	− 3	−10	1·92	2·89
1919·1	−1	+14	+ 3	+2	+ 6	−14		
1920·3	+4	+12	+ 3	−5	+ 4	−10		
1921·5	+5	+10	− 1	−2	− 4	−10		
1922·7	+4	+ 9	− 3	+5	− 4	−12		
1923·9	−3	+10	− 2	−1	− 2	− 7	1·87	2·47
1925·1	−2	+ 5	− 2	−6	− 5	0		
1926·3	0	− 1	− 9	−1	− 7	+ 2		
1927·5	−2	− 2	− 2	−3	− 2	+ 2		
1928·7	0	−10	− 4	+1	− 1	+ 6		
1929·9	−1	− 8	− 3	−1	− 1	+ 6	1·68	1·89
1931·1	+1	− 9	+ 5	0	+ 1	+ 9		
1932·3	0	− 7	+ 2	+2	+ 1	+ 8		
1933·5	−2	− 3	+ 5	0	+ 5	+ 3		
1934·7	+2	− 5	+ 6	0	+ 7	+ 5	1·68	1·78
1935·9	0	− 5	+ 6	+1	+ 6	+ 3		
(1937·1)	−1	− 2	+ 9	+1	+ 5	+ 1		

The last interval is incomplete. The residuals for y in the first 1·2 years are so irregular that their standard error has been given separately.

We see that the standard error of a mean over an interval would be 0·5 to 1 unit, and random observational error is not available as an explanation of either the fluctuation of the means already noticed or of the variation of c and c' now seen to be superposed on it. Again, the mean observational error of an average over 0·3 year would be 1 to 2 units, and cannot explain

the departure of our λ from 1. This must be regarded as an abnormally large error of some kind, though it is still not impossible that a correlation between adjacent errors has been introduced by smoothing and interpolation, and consequently that these estimates are too low. On the whole $a+b'$ and $b-a'$ are small. This is in accordance with the usual theory, according to which the motion is in a circle and therefore the motions in x and y should have the same amplitude and phases 90° apart. This may be examined a little more closely by taking the sums a^2+b^2, $a'^2+b'^2$ and $(a+b')^2+(b-a')^2$, over 6-year periods.

	a^2+b^2	$a'^2+b'^2$	$(a+b')^2+(b-a')^2$	E
1891·5–1897·4	834	905	274	37
1897·5–1903·4	1029	971	178	22
1903·5–1909·4	1484	1574	68	13
1909·5–1915·4	2301	2650	85	15
1915·5–1921·4	947	982	121	20
1921·5–1927·4	406	407	67	10
1927·5–1933·4	356	229	51	11
1933·5–1938·1	241	179	22	8
Total	7598	7897	866	136

The column E gives the expectations of the previous column, assuming that all departures of the observed values from theory are due to the observational errors estimated. The actual amounts are about six times their expectations. The first two totals are nearly equal, and the third is small enough in comparison with them to show that the theory is a good approximation to the facts, but there are significant departures. These might be due to either an incorrect adopted free period or to random disturbances becoming appreciable within 1·2 years. A method of estimating the period from $ab'+ba'$ was devised, depending on the principle that if the period is not 1·2 years there would be systematic differences from 90° in the phase-differences given by using 1·2 years as the trial period ; but it showed only that such a departure is too small to be detected in this way. It appears therefore that real disturbances produce most of the discrepancies in the third column. From the correlation of the items in this column with those under E rather than with the amplitudes it seems likely that they have also contributed some of the estimate of the observational error.

A closer analysis may be attempted by comparing harmonic coefficients over different years. We notice from inspection, particularly of $\frac{1}{2}(a-b')$ and $\frac{1}{2}(b+a')$, which should give somewhat better estimates than either a and b or a' and b' separately, that sometimes the variation is smooth over long intervals, and then suddenly interrupted. Thus there are abrupt changes in amplitude or phase at the years indicated by 1895·1, 1897·5, 1925·1, 1928·7, 1931·1 and 1933·5. From 1904·7 to 1908·3 there is a rapid building up of amplitude, a sufficiently remarkable fact in itself since it means that disturbances produced an effect in a year comparable with the whole amplitude of the mean annual terms. From 1898·7 to 1903·5 the motion is nearly constant, suggesting that either there was little damping in this range or that the disturbances happened to be then in such phases as to balance it.

But from 1908·3 to about 1921·5 we have a steady decline in amplitude with little change of phase (though a steady variation of phase in one direction would be equally to the point). It seems reasonable to suppose that this represents a stage when the large amplitude was being steadily damped out and disturbances were doing little to regenerate it. If so we can use this range to estimate the period and damping, treating the variation as simply observational error. This is not in contradiction with the fundamental principles stated earlier. If in the pendulum problem small impulses were given at frequent intervals, so that a disturbance is nearly certain to occur in most intervals between observations, there is no alternative to discussing the whole of the data together. But if the disturbances are large and infrequent it may be possible by studying the displacements to identify ranges of time when no disturbance occurred, and then analysis of such ranges taking account only of observational error would be valid. It appears here that from 1908·3 to 1921·5 is such a range. In the first place a linear form was fitted to the phases, the weights used being inversely as the amplitudes, so that the harmonic factor was assumed to be $\cos\{(1+p)300^\circ\ t-\alpha\}$ where the unit of time is a year and p is small. This gave $p=-0{\cdot}002\pm0{\cdot}013$; whence the period is

$$1{\cdot}202\pm0{\cdot}016 \text{ years} = 439{\cdot}0\pm5{\cdot}8 \text{ mean solar days}$$
$$= 440{\cdot}2\pm5{\cdot}9 \text{ sidereal days.}$$

This agrees fairly well with the previous analysis ; it is probably somewhat better because the hypothesis of occasional large disturbances appears to agree with the facts rather better than that of frequent small ones, and because the disturbance associated with the drifting mean is more effectively eliminated.

Similarly the exponential factor was estimated by applying the method of least squares to the amplitudes over this range. It gave $k=0{\cdot}0663\pm0{\cdot}0070$, corresponding to a time of relaxation of $15{\cdot}1\pm1{\cdot}8$ years. The residuals were not noticeably systematic, and it appears that exponential damping was not far from the truth.

The Greenwich values of the variation of latitude from 1911 to 1936 have been reduced on a uniform plan and published by Dr. H. Spencer Jones.* Smoothed values to intervals of 0·05 year are given, but as the observations have been collected before smoothing into about 10 groups for each year it appeared that nothing would be gained by using all the values, and only those at multiples of 0·1 year were used. These were rounded off to one unit of 0″·01 as before.

Analysis for the annual period as before gave for the six-year intervals the following totals :—

	0·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
1912–17	−42	−61	− 68	−61	−31	+12	+37	+40	+21	− 7
1918–23	−32	−47	− 49	−39	−17	+21	+59	+59	+27	−18
1924–29	−19	−24	− 27	−29	−22	− 6	+11	+ 6	− 7	−28
1930–35	−55	−90	−103	−91	−58	− 9	+31	+48	+33	0

* Royal Observatory, Greenwich : Observations with the Cookson floating zenith telescope in the years 1927–36. . . .

The separate annual terms from these totals are

1912–17	−16	$-24\cos L - 51\sin L$
1918–23	− 4	$-21\cos L - 45\sin L$
1924–29	−14	$-10\cos L - 14\sin L$
1930–35	−29	$-21\cos L - 77\sin L.$

The variation is remarkable. That of the harmonic coefficients is larger than in those from the international data, apparently because the intervals that happen to be most convenient in this case coincide with the extreme ranges of the annual term, which seems to be genuinely variable. The constant terms also are not removed, as might have been expected from the greater homogeneity of the data and the method of reduction. They do, however, give second differences small enough to permit a certain amount of optimism about the possibility of interpolation. The mean annual terms in x, found by comparison of the coefficients, are

$$-3{\cdot}2\cos L - 7{\cdot}8\sin L,$$

with a standard error of 1·6 in each. This is in good accord with the determination from the international data. The interpolated values of the mean annual corrections are

1911	−4	1918	−2	1925	−2	1931	−4
12	−4	19	−1	26	−2	32	−4
13	−3	20	−1	27	−2	33	−5
14	−3	21	−1	28	−3	34	−5
15	−3	22	−1	29	−3	35	−5
16	−2	23	−1	30	−4	36	−6
17	−2	24	−2				

The drift is similar to that in x from the international data, though it is smaller in the second part of the range. This is connected with the fact pointed out by Dr. Spencer Jones (p. 70) that a change in the adopted mean latitudes requires an increase of 4·8 units in all international values since 1922·7. The following are the residuals after removing the mean annual term and the mean annual values :—

	0·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
1911	...	...	...	...	...	...	...	+ 9	+13	+17
1912	+20	+17	+ 5	− 9	−16	−19	−19	−14	− 7	+ 3
1913	+12	+21	+24	+20	+12	+ 1	−11	−16	−18	−16
1914	− 9	+ 3	+12	+16	+17	+14	+ 6	− 2	− 9	−13
1915	−14	−13	− 8	− 1	+ 6	+16	+24	+17	+ 6	+ 5
1916	−14	−19	−19	−15	− 7	+ 4	+12	+18	+19	+14
1917	− 1	−10	−16	−18	−13	− 4	+ 1	+ 7	+12	+16
1918	+13	+ 7	0	− 5	− 8	−10	−10	− 5	0	+ 7
1919	+11	+14	+13	+ 7	+ 1	− 1	− 3	− 7	−12	−11
1920	− 2	+ 6	+11	+13	+13	+12	+ 9	+ 6	+ 2	−11
1921	−12	− 6	− 1	+ 5	+14	+15	+11	+ 5	0	− 6
1922	−11	− 9	− 3	+ 1	− 1	+ 5	+15	+14	+ 5	− 1
1923	− 6	−10	−14	−17	−17	−11	+ 2	+ 5	+ 3	− 1

	0·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
1924	− 2	− 4	− 6	− 9	−12	−16	−14	− 8	− 3	− 2
1925	+ 4	+ 7	+ 4	0	− 2	− 4	− 7	− 8	− 4	− 2
1926	0	+ 4	+ 6	+ 2	− 1	− 2	− 2	− 2	− 1	+ 1
1927	+ 8	+12	+13	+12	+ 9	+ 1	− 1	− 1	− 4	− 7
1928	− 2	+ 4	+ 5	+ 3	+ 3	+ 4	+ 1	− 4	− 4	0
1929	+ 5	+ 9	+13	+13	+ 7	+ 7	+ 7	− 5	−13	−16
1930	−14	− 8	− 2	+ 2	+ 6	+ 8	+ 7	+ 5	+ 2	− 2
1931	− 7	−12	− 9	− 6	− 2	+ 3	+ 6	+ 9	+ 4	+ 2
1932	− 6	− 7	− 8	− 9	− 9	− 2	+ 8	+ 8	+ 4	+ 4
1933	+ 5	+ 1	− 7	− 8	− 7	− 5	− 1	+ 5	+10	+11
1934	+ 5	0	− 4	− 6	− 5	− 2	− 2	− 1	+ 4	+ 7
1935	+ 7	+ 5	+ 2	− 1	− 2	− 2	− 2	+ 1	0	− 3
1936	− 1	+ 3	+ 5							

The variability of the annual means is very prominent; we notice, for instance, the prevalence of negative values in 1923–24 and of positive ones in 1927. Harmonic analysis over ranges of 1·2 years identical with those used for the international data gave the following values :—

	c	*a*	*b*		*c*	*a*	*b*
1911·9	−1	+19	+6	1923·9	−6	+ 8	−1
13·1	+2	+19	+1	25·1	0	+ 6	−2
14·3	0	+14	+8	26·3	+4	0	−8
15·5	0	+19	+7	27·5	0	0	−4
16·7	+1	+18	0	28·7	+3	−10	+2
17·9	+1	+12	0	29·9	−2	−10	−3
19·1	+1	+11	+2	31·1	−2	− 8	+1
20·3	+4	+11	+5	32·3	−1	− 8	+4
21·5	+2	+11	−2	33·5	0	− 2	+7
22·7	−2	+13	−2	34·7	+2	+ 1	+4

The standard errors of one observation found by comparison with these values were mostly about 2·5 units, so that those of the annual means should be about 0·7 and of the harmonic coefficients 1·0 if the variation was random. This is clearly inconsistent with the large changes in *c*. The trend of the coefficients is similar to that of those for *x* from the international values, but here again several of the differences are larger than one might expect. The international ones are the more regular, contrary to what one might have expected. It seems that analysis of the Greenwich data will not add appreciably to the accuracy of the estimates of the free period and the damping; those already given have also had their irregularities further reduced by combining values from *x* and *y*.

The next step would naturally be to try to estimate the changes of the products of inertia needed to account for the irregular displacements. We could use the harmonic solutions simply as smoothing formulæ to give values at times 0·0, 0·3, 0·6, 0·9 and 1·2 years after the first date covered by each, and then average the value found for 1·2 years in one interval with that for 0·0 in the next. Intermediate values could then be filled in by interpolation, and the result should be much more free from short-period disturbances and observational error than the data now available. They would also stand numerical differentiation to determine the variation of the products of

inertia. This, however, hardly seems worth while until the departures from homogeneity have been more thoroughly removed by a comprehensive discussion of the international data.

The sudden changes in a, b and c, amounting to 10 units in one case, constitute a serious problem. The annual terms provide some standard of comparison, since they appear to be maintained chiefly by the seasonal change in the distribution of the air, especially the change between summer and winter in Central Asia. But this is not a small phenomenon ; it is large enough to include the climate of India as a by-product. It seems impossible that a meteorological change capable of producing an effect comparable with the mean annual effect could occur without having received special comment from meteorologists. Further, the mean annual motion of the pole is disproportionately large in relation to the actual disturbing cause, since it is magnified by the approximate agreement between the forced and free periods. A fairer comparison would be given by the displacement of the pole *of inertia* by a surface inequality of mass equal to $j \cos\theta \sin\theta \cos\phi$ per unit area, where θ and ϕ are the north polar distance and east longitude. We find for the product of inertia

$$G = \tfrac{4}{15}\pi j a^4$$

and the displacement of the pole of inertia is $-G\tau/A$, where τ is the number of days in the period of the free motion and A the moment of inertia about an equatorial axis. If this displacement is $0''{\cdot}10$ this leads to $j = 6$ gm./cm.^{2}, and the disturbance of the distribution of mass must keep the same sign over a quadrant of the sphere, reaching a maximum of 3 gm./cm.^{2}. The annual variation in Central Asia is about ten times this, but reaches such a value over only a small fraction of the quadrant. The quantity seems small in relation to the whole mass of the Earth per unit surface, but must be highly systematic to be as small as this. All changes of the Earth's body known either to observation or theory seem to have either a totally wrong time-scale or horizontal extent. Surface movements associated with earthquakes, for instance, may reach larger amounts but are extremely local. Secular changes such as may ultimately give rise to mountain building would have time scales of the order of 10^8 years, and do not appear admissible as an explanation of changes that appear in one year and are reversed in the next.

This estimate invites comparison with the changes of the Earth's rotation needed to account for the fluctuations of the motions of the Sun, Moon, Mercury and Venus discussed by Dr. H. Spencer Jones.* Inspection of his diagrams suggests that the rate of change of the Moon's mean longitude had an unexplained fluctuation during the nineteenth century through a range of about $0''{\cdot}6$ per year ; to explain this by a change of the rotation of the Earth would mean one of about $17''$ per year in the latter, which is about 5×10^{-8} of the whole rotation. This would mean a change of the moment of inertia by a similar fraction, or of the equatorial radius by 15 cm. This is rather larger than the abrupt changes of the polar motion suggest, but corresponds to a much longer interval of time. If genuine it might possibly be detected

* *M.N.*, 99, 541–558, 1939.

by analysis of tidal records over long periods, since such a fluctuation would disturb the ellipticity term in the Earth's gravitational field. I should not myself be inclined to consider such changes of the Earth's form as proved by either set of data until confirmatory geophysical evidence is obtained, and they would not be a complete explanation until they themselves are explained. At present it seems necessary to leave the matter in this rather unsatisfactory state.

The damping of the free motion has an obvious qualitative explanation, since it must be associated with an oceanic tide. This will combine with the semidiurnal tide and contribute to the dissipation in shallow seas. A portion of the reaction from this will have the same period as the free motion and will tend to damp it. I have previously estimated the consequent dissipation of energy in the free motion as of the order of $5 \times 10^{23} l^2$ ergs per second, where l is the amplitude of the free motion in radians.* Recalculating this I get $10^{23} l^2$. The estimate of the energy in the motion, however, is seriously wrong. For a rigid body, in the usual notation of dynamics, the kinetic energy would be given by

$$2T = A\omega_1^2 + B\omega_2^2 + C\omega_3^2,$$

where A and B may be taken equal in this problem. The standard of comparison would be the same body after damping so as to be in steady rotation Ω about a principal axis. Then we can find Ω by noticing that the resultant angular momentum G is an invariant for internal changes and would not be affected by dissipation. Hence

$$G^2 = A^2\omega_1^2 + A^2\omega_2^2 + C^2\omega_3^2 = C^2\Omega^2$$

and the standard energy T_0 is $\frac{1}{2}C\Omega^2 = G^2/2C$. Then

$$\begin{aligned} 2(T - T_0) &= (A - A^2/C)(\omega_1^2 + \omega_2^2) \\ &= \frac{A(C-A)}{C}\Omega^2(l^2 + m^2) \\ &= 10^{34}(l^2 + m^2) \text{ ergs} \end{aligned}$$

nearly. The time of relaxation for the energy is therefore

$$\tfrac{1}{2} \times 10^{11} \text{ sec.} = 1700 \text{ years.}$$

That for the amplitude would be twice this, or 3400 years. This is about 5000 times shorter than I obtained previously, a factor $C/(C - A)$ having got into the numerator instead of the denominator. Nevertheless it is still 200 times as long as the observed time.

The calculation is naturally very rough, but it is difficult to see any error that could account for a factor of 200. One that needs special mention is the assumption in calculating the rate of dissipation, that the contributions of the semidiurnal and 14-monthly tides to the currents are in the ratio of the rates of change of the equilibrium vertical displacements. This might apparently be far from the truth when the periods are so different. But it is

* *M.N.*, 80, 317, 1920.

easy to show that in a tidal problem, even with allowance for dissipation and the rotation, the currents do contain the rate of change of the equilibrium tide as a factor. The standard of comparison was the rate of dissipation in the semidiurnal tide, and it appears that rotation and dissipation are already fully effective in controlling this, since the equilibrium potential energy would be dissipated in something of the order of a period. Consequently, though the phase lag in the 14-monthly tide might conceivably be smaller than in the semidiurnal one, it is very unlikely to be larger, and the estimated dissipation in it will not be too low.

To put the matter in another way, it is useless to assume more damping in the 14-monthly tide in the ocean than would give it a time of relaxation comparable with its period if it was left to itself. To assume too much damping in the ocean would reduce oceanic motions of this period so much that the ocean would rotate with the Earth, and the damping of the motion of the Earth by the reaction would be reduced. Thus at the best we could take tidal friction as damping out an oceanic motion in about a year ; but the energy in the motion of the Earth will be about 2000 times that in the ocean, on account of the greater moment of inertia, and must therefore persist for some 2000 years.

On the other hand, the alternative to accepting the observed time of relaxation would be to say that the change of amplitude from 1908 to 1921 was due to a series of small disturbances all tending to reduce the actual motion. But the general tendency of such disturbances would be to increase the oscillation of the pole, even though there would be limited ranges of decrease, and there were actually a considerable number of order $0''{\cdot}10$ in 40 years. If then the time of relaxation was as long as 3000 years it becomes difficult to see how they could have failed in 3000 years to work up an amplitude much larger than occurs.

It looks therefore as if we must seek for the explanation, not in the oceanic effects, but in an imperfection of elasticity in the body of the Earth. It has already been seen that such imperfection could not explain the secular acceleration of the Moon. If it was due to plasticity it would make the time of relaxation for the 14-monthly period much too short. The same applies to elastic afterworking.* This in its simplest form requires two time moduli τ and τ' to express it, where $\tau' > \tau$, and the phase lag in a forced motion would be α, given by

$$2\alpha = \tan^{-1}\gamma\tau' - \tan^{-1}\gamma\tau,$$

where $2\pi/\gamma$ is the period. The measured rigidities found for disturbances of very long and very short periods would be in the ratio τ/τ'. For the Earth's material we must suppose τ and τ' long compared with a day, and the second alternative mentioned in the paper quoted is the correct one. Then α may be small for semidiurnal motions on account of the two angles both approximating to $\frac{1}{2}\pi$; but it might be considerable for longer periods. For very long periods of course it again approaches 0 since both angles tend to zero. But it seems possible that with τ and τ' of the order of some years we might

* *Proc. Roy. Soc.*, A, **138**, 296, 1932.

explain the relaxation by elastic afterworking in the body of the Earth without introducing inconsistency with either the data about tidal friction or those about the strength of the materials concerned. The 14-monthly period could be in the range that is sensitive to this kind of imperfection of elasticity, though damping due to it would tend to zero both for very long and very short periods.

The estimate of the free period leads to the value for Love's number k :

$$k = 0{\cdot}288 \pm 0{\cdot}008,$$

agreeing with that already given from Pollak's estimate *, but an estimate of uncertainty is now available. We notice that in (4) the harmonic terms are independent of the damping, which in any case is small within a period, and it is likely that the correction for damping will be small. Rosenhead † got the following theoretical values assuming a uniform core and shell with average properties, the core being taken as of zero rigidity :—

Period	Static	Fortnightly	Semidiurnal
Ocean neglected	0·364	0·364	0·364
Ocean depth 7260 feet	0·270	0·288	0·119
Ocean depth 29,040 feet	0·270	0·279	0·799

The great differences for the semidiurnal tide are due to the presence of free periods in the neighbourhood. Probably if friction was taken into account they would be much smaller. Comparison of the static and fortnightly values shows that Rosenhead's model gives about the right value of k for the 14-monthly period, and that this does not depend greatly on the adopted depth of the ocean provided it is not altogether neglected.

The presence of damping may help to reduce a difficulty found by Rosenhead. My discussion of the explanation of the annual period and his further discussion, assuming an equilibrium theory for the effect of surface pressure over the oceans, both gave too large an amplitude and a considerable error in phase. This appeared to be removed by Rosenhead by means of a result of Nomitsu and Okamoto ‡, who found that along the coast of Japan the variation of sea-level was about three times what the effect of air pressure would explain directly. Taking the disturbance of the ocean by meteorological effects as three times the equilibrium value, Rosenhead got a good agreement for the annual terms in the variation of latitude, but agreement for k was made worse. It appears that the extra effect found by Nomitsu and Okamoto is due to changes of salinity. On an equilibrium theory these would be compensated by a change of sea-level in such a way as to leave the pressure at a fixed point unaltered, and the previous equilibrium theory of the annual variation of the distribution of mass over the oceans would stand. But if the annual tide in the ocean is subject to considerable damping there will be large differences of phase, which might account for the discrepancy.

* *The Earth*, p. 237, 1929.
† *M.N., Geoph. Suppl.*, **2**, 195–196, 1929.
‡ *Sci. Mems. of Kyoto Imp. Univ.*, A, **10**, 125–161, 1927; **15**, 123–129, 1932.

Summary

The international data on the variation of latitude have been analysed in accordance with the idea that the free motion is a damped one maintained by disturbances at irregular intervals of time. The results suggest a free period of 1·202 ± 0·016 years and a time of relaxation of 15·1 ± 1·8 years. There are notable irregularities in the motion, both in the harmonic coefficients and in the means taken over 1·2-year intervals, which occur in both the international and the Greenwich data. If of geophysical origin they imply changes in the products of inertia that are difficult to explain. The damping is too great to be attributed to friction in the sea, but might be due to elastic afterworking in the body of the Earth.

PHOTOMETRIC OBSERVATIONS OF $H\alpha$ IN THE SOLAR SPECTRUM

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(Communicated by the Director, University Observatory, Oxford)

(1) *Introduction.*—The factors affecting the formation of a spectrum line in a stellar atmosphere may be divided into two related groups. One consists of general factors such as opacity and temperature which are characteristic of the atmosphere as a whole, and the other of special features characteristic of particular lines or series.

In the case of $H\alpha$ two special features may be present. One is the existence of uncompensated cycles of atomic transitions, and the other is intermolecular Stark effect.

The first is possible because the Balmer series is not the principal series of hydrogen, and hence an atom excited into the three-quantum state may return, not to the two-quantum state, but to the ground state. If the reverse process does not occur equally often an uncompensated cycle of transitions will be set up.

Unsöld has suggested * that this state of affairs holds in the formation of the Balmer series with the possible exception of $H\alpha$. He suggests further that the effect (as far as the mathematical theory of line-formation is concerned) is the same as if the lines were formed by pure absorption. If this were the mode of formation of the lines then they would disappear completely at the limb of the Sun, and a test of the hypothesis would be easy.

However, this second suggestion does not seem to be correct.† The atoms which are de-excited by superelastic collisions differ from those which return through an uncompensated cycle in that their energy is returned to the general radiation field in the frequency of the absorption line, while the energy of an atom taking part in a cyclical process is transferred to an entirely different frequency.

The equation of transfer which describes this state of affairs is :

$$\cos\theta \frac{d\mathcal{J}_\nu'(\theta)}{d\tau} = (1+\eta)\mathcal{J}_\nu'(\theta) - (1-\epsilon-\delta)\eta\mathcal{J}_\nu' - \epsilon\eta\mathcal{J}_\tau - \mathcal{J}_\tau, \qquad (1)$$

in which ϵ is the proportion of atoms de-excited by superelastic collisions and δ is the proportion taking part in the cyclical process.

The observations will be discussed on the basis of this equation.

The second important peculiarity of the Balmer series is that its lines may be broadened by the Stark effect due to momentary molecular inter-

* *Physik der Sternatmosphären*, p. 288, 1938.

† I am indebted to Professor Plaskett for pointing this out.

action in the atmosphere. In a previous paper* calculations, based on the work of Pannekoek and Verweij, of the centre-to-limb variations of the Balmer series were made with the object of comparing the results with the observations of Redman† on eclipsing binaries. He had found a decrease in equivalent width towards the limb, and although the calculated intensities also showed this effect the results did not form a real test of Verweij's theory since the agreement was only qualitative and such a decrease would be predicted by any theory in which η increased with depth. The results were also confused by uncertainties in the calculated opacity. It was also thought (following Unsöld's suggestion) that the effective value of ϵ might be large, but it was impossible to do more than guess its value.

The uncertainties of the present situation can only be cleared away by accurate observations of the centre-to-limb variations of line contours from which it may be possible to deduce the mode of formation of the lines, and the distribution of absorbing sources, without invoking an uncertain theory of stellar opacity.

It was with these objects in mind that the present programme of observations of the centre-to-limb variations in the contour of $H\alpha$ was undertaken.

It was originally intended to combine both an account of the observational methods and a theoretical discussion of the results in a single paper, but it has now been decided to publish the observations separately with only a brief preliminary discussion and to defer their thorough discussion until a more convenient time.

(2) *The Observations.*—The work was carried out with the Oxford Solar Telescope‡, which has an aperture of 12 inches, a focal length of 62 feet and gives a solar image 186 mm. in diameter. The spectroscope is of the Littrow form§ with two 60° prisms and one 30°. It has the same aperture ratio as the telescope and gives a dispersion of 2·06 A.U. at $H\alpha$ and 1·47 A.U. per mm. at the sodium D lines. The combined collimating and camera lens is designed with three of its four faces concave to the slit so that the real images formed by reflection off these faces can be diaphragmed off, thus reducing the scattered light in the instrument to a very low value.

The plates used were Ilford Rapid Process Panchromatic, a choice which was rather unsuitable because of their rapid loss of sensitivity immediately beyond $H\alpha$. However, the necessity of overcoming this defect led to the development of new methods of drawing characteristic curves and of drawing the continuous spectrum background which turn this rapid change to advantage.

The means used to produce known reductions in the intensity of the incident light was a platinum-on-quartz step wedge by Zeiss. This had five sputtered strips and was mounted on a web which left free a small step

* "The Stark Effect of Hydrogen and Centre-to-Limb Variations in the Contours of $H\beta$, $H\gamma$ and $H\delta$," *M.N.*, **99**, 17, 1938.

† Redman, *M.N.*, **96**, 488, 1936.

‡ H. H. Plaskett, "The Oxford Solar Telescope and Hartmann Tests of its Performance," *M.N.*, **99**, 219, 1939.

§ An account is given in *Engineering*, 1939 March 24.

of clear quartz at top and bottom of the sputtered strips, giving seven steps in all. It was originally intended to use this wedge before the slit of the solar spectroscope, but it was found that the wedge was sputtered on a piece of quartz which was not plane (presumably in order to avoid interference effects). This departure from parallelism in conjunction with the small aperture ratio of the solar spectroscope was sufficient to throw the incident beam out of collimation, and it was therefore decided not to use the wedge on the solar spectroscope, but to calibrate the plates in the laboratory in an auxiliary spectroscope having a dispersion of 117 A.U. per mm. at 6200 A.U.

Two solar exposures were made on each plate using a red filter (Wratten, No. 71) to remove all but a few hundred angstroms of spectrum, and a web having four spaces (see Plate 1, fig. 1) to split up the spectrum parallel to the dispersion in order to reduce scattered light. The total length of slit used was 8 mm., and the end space of the web was twice the height of the others. The limb spectra taken with a radial slit were exposed by guiding the Sun's limb on a point half-way up this end space. Plate E15 was taken, using as a guide a cardboard diaphragm with two concentric circles of diameters 188 and 184 mm. drawn on it and a series of apertures cut at various values of $\cos\theta$ along a diameter.

After exposure in the solar spectroscope the plates were given four calibration exposures through the wedge and the red filter in the laboratory, using as the light source a magnesium carbonate block illuminated by a six-volt lamp. In order to give a greater range in opacity than would be afforded by a single wedge exposure, and at the same time to give a denser distribution of points on the characteristic curve, the incident intensity was varied as between different calibration exposures by varying the slit width. A standard set of slit widths was taken for all the plates, and for each separate plate the lamp was fixed at a suitable distance from the block. Wave-lengths were determined by adding four exposures from a helium lamp with the same standard slit widths.

After the exposures were completed the plates were stored for 24 hours so that the latent image might complete its initial rapid fading, and they were then brush-developed for 8 minutes at 65° F. in Parkhurst two-solution developer.

Particulars of the eight plates selected for measurement are given in Table I.

(3) *Measurement and Reduction.*—The tracings were made with the Oxford microphotometer *, which has two ranges—a low magnification range covering 7 cm. of plate, and a high magnification covering 7 mm. Each strip of spectrum was traced on both ranges, the one giving the centre accurately, and the other all the wings of the line. The tracings showed immediately that the fitting of the continuous spectrum background was likely to be difficult for (see Plate 1, fig. 2) the plate sensitivity fell off rapidly and in a non-linear manner over the range of the tracings.

* H. H. Plaskett, "A Moll-type Microphotometer and its Performance," *M.N.*, 95, 160, 1934.

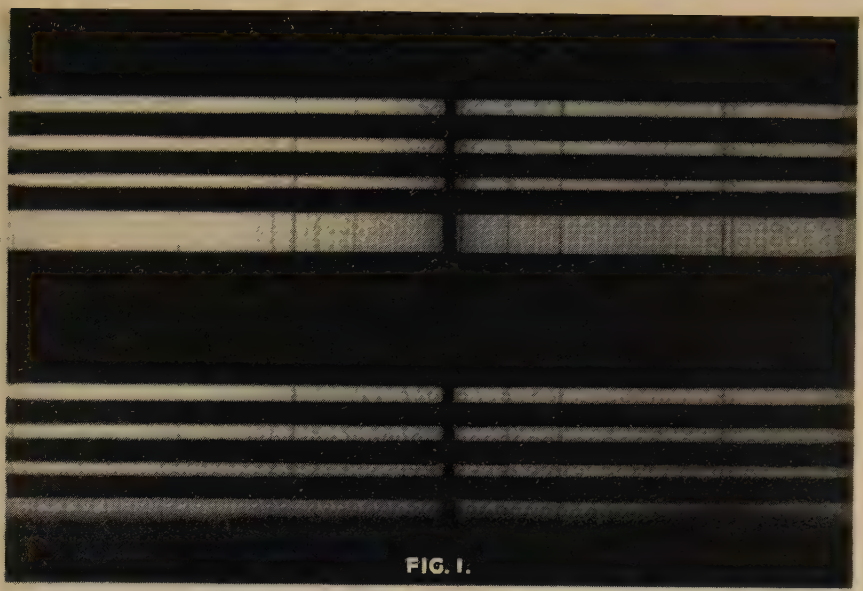


FIG. 1.

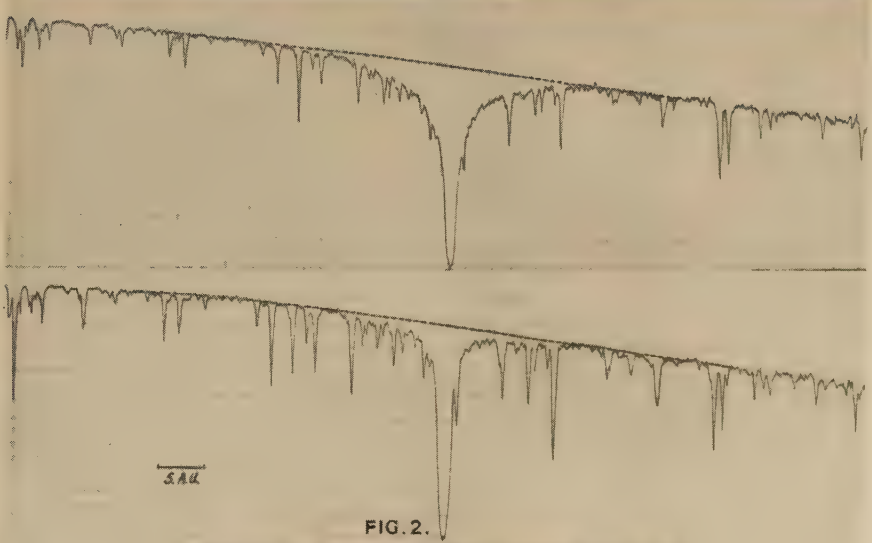


FIG. 2.

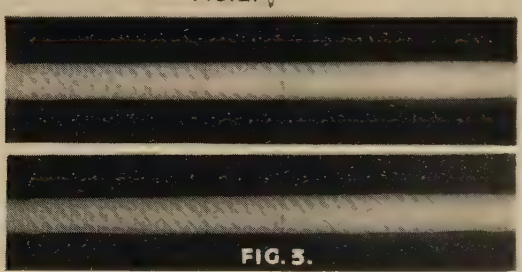


FIG. 3.

Fig. 1.—Hα in solar spectrum. Top: centre. Bottom: limb.
Fig. 2.—Tracings of Hα. Top: centre. Bottom: limb.
Fig. 3.—The Sodium D lines. Top: 120°. Bottom: 130°.

TABLE I

Plate No.	Date and Time	Subject	Solar and Calibration Exposure Time	sec ζ	Distance of Lamp from Block
			m.		cm.
E11	1938 Nov. 2·563	Limb	5·0	2·77	10
	Nov. 2·569	Centre	5·0	2·82	
E12	Nov. 2·577	Limb	5·0	2·90	10
	Nov. 2·589	Centre	5·0	3·04	
E13	Nov. 2·603	Limb	5·0	3·26	10
	Nov. 2·609	Centre	5·0	3·37	
E15	Nov. 2·638	cos θ = ·741	5·0	4·11	10
	Nov. 2·645	,,	5·0	4·35	
E27	Nov. 5·492	Prec. Limb	2·0	2·59	4
	Nov. 5·496	Foll. Limb	2·0	2·59	
E28	Nov. 5·498	Foll. Limb	3·0	2·60	6
	Nov. 5·503	Prec. Limb	3·0	2·60	
E29	Nov. 5·513	Prec. Limb	2·0	2·62	4
	Nov. 5·518	Foll. Limb	2·0	2·63	
E32	Nov. 5·540	Foll. Limb	3·0	2·72	6
	Nov. 5·545	Prec. Limb	3·0	2·75	

Slit widths: Solar exposures, 0·05 mm.
Calibration exposures, standard set, 0·20, 0·35, 0·50, 0·65 mm.

The opacities on the tracings were measured, and the wave-lengths fixed by laying down a linear scale on each tracing and drawing a curve of errors from the wave-lengths listed in Rowland's table.

Three determinations were then necessary in order to convert these opacities into residual intensities.

These were :

(*a*) The determination of the transmissions of the steps of the wedge.
(*b*) The estimation of the continuous spectrum backgrounds.
(*c*) The determination of the characteristic curves of the plates.

(a) *The Calibration of the Wedge.*—This was accomplished with the aid of the set of 7 diaphragms used by H. H. Plaskett in his work on the magnesium *b* lines in the solar spectrum.* These diaphragms each contained 21 symmetrically and identically placed equal holes, particulars of which are given below.

* "The Formation of the Magnesium *b* Lines in the Solar Atmosphere," *M.N.*, 91, 870, 1931.

Table II

Diaphragm No.	Diameter of Holes	Ratio of Areas
	mm.	
1	3·637	1·000
2	2·734	0·565
3	2·129	0·343
4	1·707	0·220
5	1·290	0·126
6	1·038	0·0814
7	0·866	0·0567

Thus if a diaphragm is placed in a definite position normal to a parallel beam of light whose intensity at each point of its section is constant with time, the amount of light transmitted will be proportional to the areas of the holes.

These conditions were realised in practice by placing the diaphragms in a holder in the collimated beam before the prism of the laboratory spectroscope. As before, the light source was an illuminated area on the magnesium carbonate block. The lamp was carefully screened so that no direct light could enter the slit of the spectroscope, and the work was carried out in a room with blackened walls. An ammeter and rheostat were used to maintain the current in the lamp exactly constant. The calibration of the wedge was thus carried out under the same conditions as applied when it was used.

Six exposures of seven strips each were made on each calibration plate. Of these two were wedge exposures, one with Diaphragm No. 1 and the other with Diaphragm No. 2. The other four were taken through the web belonging to the wedge and using four diaphragms chosen so that their transmissions straddled the approximately known transmissions of the wedge steps which were to be determined from the plate.

The illumination along the slit was not quite uniform and so the web spectra varied slightly in opacity from strip to strip, but for any given strip of spectrum a curve could be drawn from the four web spectra relating incident intensity to opacity for this strip, and from this curve the transmission of the corresponding step of the wedge could be read off. This method of reduction by corresponding strips eliminates errors due to variation of intensity along the slit.

The lines produced by the laboratory spectroscope were not straight, but had a curvature which gave a maximum displacement between the middle and the ends of the line of about 0·3 mm. In a region where the plate sensitivity was sensibly constant this would be of little importance, but at $H\alpha$ where the plate sensitivity is varying rapidly a displacement of this amount produces a large change in opacity. However, it is well known * that the transmission of platinum changes only slowly with wave-length, and this fact saves the situation. The wave-length spectra were put on at the top and bottom edges of the plate with the ends of the slit, and hence a line ruled across the plate through a chosen wave-length in the helium spectra will

* Kienle, *Zeits. f. Ast.*, 4, 282, 1932.

pass through approximately the same wave-length in the end strips of the wedge and web spectra, but will meet the middle strips at points about 0·3 mm. (*i.e.* about 35 angstroms) to the blue of this wave-length. If now we take a tracing perpendicular to the strips through the same wave-length in the helium spectra and use the measured opacities to calibrate the wedge, the effect will be that the middle steps are calibrated at a wave-length 35 A.U. bluer than the end steps, but the error will be negligible because of the slow variation of the transmission of platinum.

The wedge was calibrated in this way at three wave-lengths, namely, at the helium line 5876, and at points 4 mm. and 5 mm. beyond this line (*i.e.* at 6344 and 6461). A slight extrapolation to *Hα* at 5·847 mm. beyond this helium line then gave the calibration there. The results from a total of 24 plates were as follows :—

TABLE III

Step No.	Transmissions (and Root Mean Square Errors)		
	5876	6344	6461
(1) (clear quartz)	0·864 ± ·025	0·877 ± ·033	0·864 ± ·039
(2)	·635 ± ·021	·612 ± ·018	·618 ± ·042
(3)	·423 ± ·011	·397 ± ·023	·384 ± ·018
(4)	·284 ± ·008	·273 ± ·015	·274 ± ·015
(5)	·167 ± ·004	·158 ± ·005	·169 ± ·010
(6)	·106 ± ·003	·104 ± ·006	·111 ± ·008
(7) (clear quartz)	0·909 ± ·017	0·914 ± ·023	0·900 ± ·048

The 4 per cent. difference in transmission between the two clear quartz steps at once aroused suspicion of the whole calibration, but these doubts were set at rest by fixing the wedge in the microphotometer and taking a tracing across the steps, when it was found that their transmissions were not in fact equal. It will be noticed that these end steps are less well determined than the others, and in the reduction of the solar plates only the middle five steps were used.

For the determination of the spectroscopic pattern it was necessary to know the transmissions of the wedge at the sodium D lines. The final values adopted there and at *Hα* were :

TABLE IV

Transmissions

Step No.	1	2	3	4	5	6	7
D lines	·868	·635	·423	·284	·167	·106	·906
Hα		·610	·381	·271	·160	·108	

(b) *The Estimation of Continuous Backgrounds.*—In order to overcome the difficulties introduced by the variation of plate sensitivity at *Hα* a series of artificial backgrounds was constructed. This was done by focussing the

light from a ribbon filament lamp on to the slit of the solar spectroscope, diaphragming the beam to $f/62$ and making sure that the alignment of the apparatus was exact. The light from this system was passed into the spectroscope through the same red filter (Wratten, No. 71) as was used for the solar exposures, and its spectrum photographed, using a series of slit widths.

The variation in background is due in effect only to the variation of the plate sensitivity, and hence tracings of the continuous strips obtained in this way will give a series of backgrounds essentially the same as the true solar background for different mean densities. These tracings were drawn out on transparent paper, which when stuck on a piece of plate-glass gave a scale which could be placed over a tracing in order to show how the continuous spectrum should be drawn in.

(c) *The Determination of Curves of Blackening.*—A characteristic curve relates opacity of the strips of the calibration spectra measured exactly at $H\alpha$ to the logarithm of the incident intensity. Because of the rapid variation of plate sensitivity it is necessary to fix the position of measurement in each strip with the greatest accuracy, and because of the curvature of the lines it will not be sufficient merely to take tracings across the strips through the wave-length of $H\alpha$ in the helium spectra, because this would introduce serious error into the measures of the middle strips.

In addition, the slit of the laboratory spectroscope was slightly inclined to the edge of the prism, and thus in order to draw the curve of blackening in the ordinary way it would have been necessary to use the edge of the plate as a datum line from which to estimate this tilt. This procedure was rejected as hopelessly inaccurate, and a new method of drawing the characteristic curves, which depended for its success on the rapid variation of sensitivity, was devised.

In drawing the curve we have to take account of errors arising from the curvature and asymmetry of the spectrum lines and from the non-uniformity of illumination along the slit. Between the end of the line and its middle these errors will be considerable, but between neighbouring strips of calibration spectrum, and especially for the middle ones where the effects mentioned are changing slowest, they will be very small. Thus a line drawn through the same wave-lengths in the top and bottom helium spectra, although not a reliable datum for all the strips, will pass through almost exactly the same wave-length in neighbouring strips. Tracings taken along these strips will show a sharply rising curve with a kick on it at the position of the ruled line.

If T_n and T_{n+1} are the transmissions of a pair of neighbouring strips, then a pair of points at the same distance from the ruled line, one in each strip, will correspond to incident intensities whose logarithms differ by the constant $\log T_n/T_{n+1}$, which is characteristic of this pair of neighbouring strips. Further along these strips we can obtain a second pair of points whose lower member coincides with the upper member of the first pair. By repetition of this process we can obtain a complete curve of blackening from each neighbouring pair of strips.

The essential assumption involved is that the curve of blackening does not change over a range of about 300 A.U., and this is verified by the internal evidence that curves derived from the same strip-pair on the same plate but with different opacities (*i.e.* the curve of blackening derived from the one refers to a higher wave-length than the other) agree within the errors of measurement.

The method also provides an excellent test of the accuracy of the calibration of the wedge, since any error in this will be magnified 6 or 7 times between the beginning and the end of a characteristic curve and will reveal

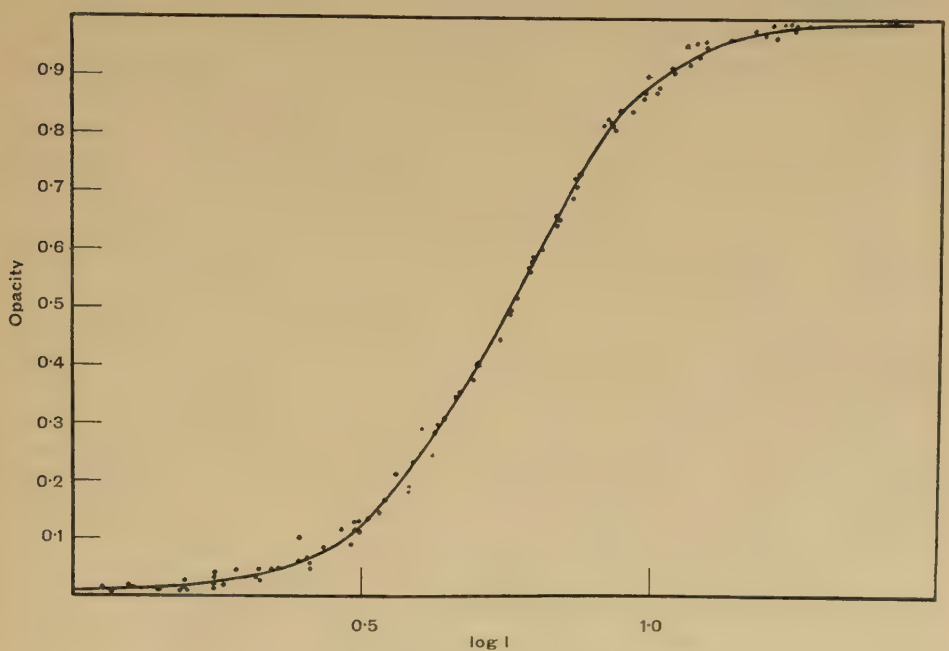


Fig. 4.—*Curve of blackening for Plate E 28.*

itself as a systematic difference in gradient between curves derived from different strip-pairs. No trace of such a difference was found.

The values for the ratios of transmissions were :

Table V

Pair	(2, 3)	(3, 4)	(4, 5)	(5, 6)
Log ratio	·204	·148	·229	·171

Thus instead of having only 20 or 30 points from which to determine the characteristic curve of a plate, we had 16 curves each containing 6 or 7 points with an internal check on their reliability. The final curve was derived by moving the component curves bodily until the least scatter of points was obtained (see fig. 4).

It was now possible to convert the opacity measures from the tracings into the corresponding values of log$I$. The artificial backgrounds then

gave log$I$ for the continuous spectrum, and then the residual intensities could be determined. In applying the continuous backgrounds the high- and low-magnification tracings of the same strip, which in every other respect had been reduced independently, were treated as a pair, and the background opacity from the low-magnification tracing transferred to its fellow. This was necessary since the high-magnification tracings did not include all the wings of the line.

The methods by which these contours were combined were worked out in the light of information obtained as a result of the determination of the instrumental pattern of the spectroscope, and a description of the method by which this determination was made will now be given.

(4) *The Determination of the Instrumental Pattern of the Spectroscope.*— If an infinitely narrow line is fed into any spectroscope it is distorted into a contour of finite width which is characteristic of the instrument, and in order to eliminate the effects of this instrumental distortion it is necessary to determine this characteristic instrumental pattern.

At the suggestion of Professor H. H. Plaskett the D lines of sodium in absorption were used for this determination, of which a preliminary account has already been given elsewhere.*

The principle of the method is to feed into the instrument a beam of light diaphragmed to the same aperture ratio as for the solar plates, through a tube containing sodium vapour at a known temperature, and to compare the observed line contours with those that can be calculated from the temperature and length of the vapour column. The total number of atoms which are operative will be calculated from the observed doublet ratio of the lines, and the amount of scattered light from the difference between the observed equivalent widths and those calculated from the theory.

The light source was a ribbon filament lamp taking a direct current of 22 amperes and burning at about 2600° K. The light was passed through an evacuated pyrex tube 25 cm. in length with plane ends, and which contained a little solid sodium. This was heated electrically in an asbestos enclosure, whose temperature was measured by a mercury in glass thermometer reading up to 400° C. After passing through the tube the light was focussed on the slit of the spectroscope by means of a lens of 8 inches focus diphragmed to $f/62$. An orange filter was placed immediately next to the diaphragm. The apparatus was all mounted on heavy wooden blocks clamped to a base-board, and the lamp, lens and diaphragm were mounted so that they could be adjusted in all their degrees of freedom to enable the paramount condition of exact alignment to be satisfied. In practice this condition was secured by ensuring that the collimating lens was filled, for, since the aperture ratio was correct, this implied that it was only just filled.

A second important condition which had to be satisfied was that the tube should be maintained at a uniform temperature over its length, and that this temperature should be constant throughout each exposure. Some

* "The Use of the D lines of Sodium in Absorption for the Determination of Spectroscopic Pattern," *Observatory*, **62**, 231, 1939.

care was needed in the design and construction of the heater to ensure that this condition should be satisfied. The heater consisted of a double-walled asbestos enclosure, and the tube, enclosed in copper sheet, was placed in the inner compartment. Other shields were arranged so as to minimise heat losses from the ends of the tube which would lead to the deposit of opaque films of solid sodium there.

In using the apparatus it was found best to maintain the temperature of the tube by heating only the air space between the compartments, and in this way the temperature could be maintained steady within a degree on

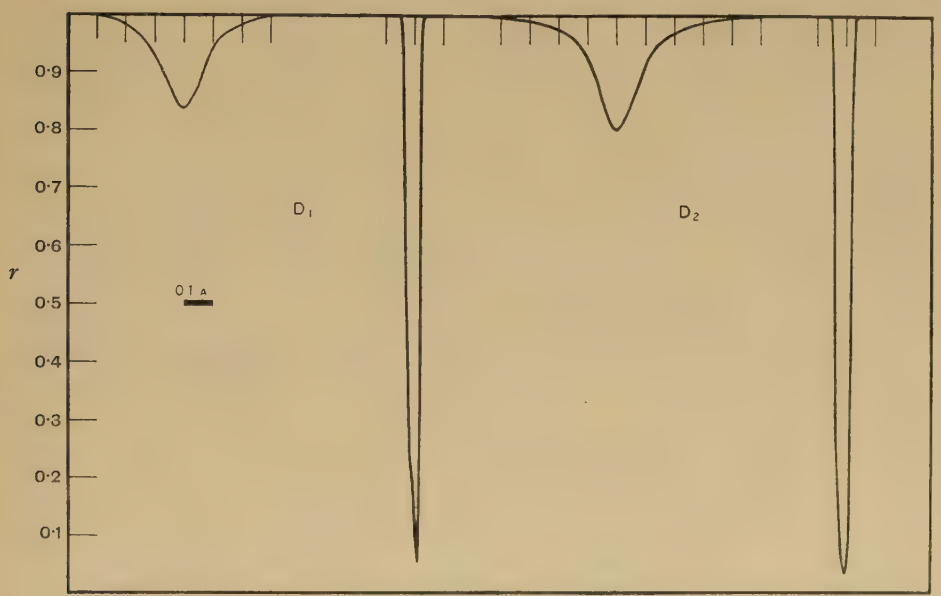


Fig. 5.—*The distorted and true contours of the D lines at* 130° *C.*

either side of the desired value, simply by switching the current on and off.

Using a slit width of 0·025 mm. it was found possible to photograph the D lines easily in 30 minutes at temperatures as low as 102° C. (corresponding to only 4×10^9 atoms per c.c.), but for the actual determination of spectroscopic pattern a temperature of 130° C. was used (see Plate 1, fig. 3).

Five plates giving a total of 10 spectra were taken at this temperature and calibrated in the same way as the solar plates. The wave-length scale was taken as linear, using the positions of the lines themselves as reference points. The curves of blackening were derived by the ordinary method of taking tracings perpendicular to the strips of the calibration exposures, with an allowance for non-uniformity of the calibrating source along the slit of the laboratory spectroscope derived from plates taken at the time of calibration.

The contours derived from these 10 spectra are given in the following table and illustrated in fig. 5 :—

TABLE VI

Residual Intensities

	D_1, 5895·930	D_2, 5889·963
0·00	0·837	0·803
·05	·880	·840
·10	·930	·920
·15	·964	·955
·20	·980	·971
·25	·991	·9815
·30	·996	·9895
·35	·998	·994
·40	0·9995	·995
·50	1·000	0·997
0·60	1·000	1·000
Equivalent widths	0·0333 ± ·0029	0·0440 ± ·0043

These mean contours were derived from the individual observations by taking the mean residual intensities at selected wave-lengths in the wings and at the centre, and the mean wave-lengths of selected residual intensities in the cores. The average mean square error in residual intensity was ·004 and in wave-length ·015 A.U.

In order to calculate the true contours of these lines the number of atoms which are operative must be determined. For this case of pure scattering we have : *

$$r = e^{-k(v)\times 25}, \tag{2}$$

where

$$\frac{k(v)}{k_0} = e^{-v^2} - \frac{a}{\sqrt{\pi}}\{1 - 2vF(v)\} \tag{3}$$

and

$$F(v) = e^{-v^2}\int_0^v e^{z^2}dz,$$

where v is given by

$$v = \frac{\Delta\lambda}{\lambda_0}c\sqrt{\frac{m}{2kT}}, \tag{4}$$

m being the mass of a molecule of sodium vapour.

Also

$$\frac{k_0}{N} = \frac{2\pi^{3/2}e^2f}{mc\Delta\omega_D} \quad \text{where} \quad \Delta\omega_D = \frac{2\pi}{\lambda_0}\frac{1}{\sqrt{\frac{m}{2kT}}} \tag{5}$$

and

$$a = \frac{\gamma}{\Delta\omega_D}.$$

In our case

$$\gamma = 0{\cdot}63 \times 10^8 \text{ sec}^{-1}, \dagger \qquad \frac{a}{\sqrt{\pi}} = 6{\cdot}294 \times 10^{-3}$$

* See Unsöld, *Physik der Sternatmosphären*, p. 159, 1938. † Unsöld, p. 275.

and $$f_{D_1}=0{\cdot}325, \qquad f_{D_2}=0{\cdot}650.$$

These are merely the expressions for the residual intensity including radiation damping and Doppler effect. N is the number of atoms per c.c. producing the line, and f is its oscillator strength.

$1-2vF(v)$ has been calculated by Mitchell and Zemansky*, and v is immediately calculable from equation (4).

Now the D lines have a hyperfine structure which must be taken into account.† $D1$ is a doublet with a separation of 0·021 A.U. and an intensity ratio of 2 : 1. The corresponding values for $D2$ are 0·023 A.U. and 2·5 : 1.

In computing the contour of $D1$ we take it to be the resultant of two blended lines with oscillator strengths 0·216 and 0·108 and having the given separation.

The original formulæ therefore require some modification :

$$k_1(v)=k_1'(v')+k_1''(v'') \quad \left(\text{where} \quad v'=\frac{\Delta\lambda+{\cdot}021}{\lambda_0}c\sqrt{\frac{m}{2kT}}=v+\delta, \quad \text{say,}\right)$$

$$=\left(\frac{k_0}{Nf}\right)\left\{{\cdot}216\left(\frac{k(v)}{k_0}\right)+{\cdot}108\left(\frac{k(v+\delta)}{k_0}\right)\right\}, \qquad (6)$$

where the expressions in curved brackets denote the functions defined by equations (3) and (5).

Then

$$\log_{10}\left(-\frac{\log_{10} r}{N}\right)=1{\cdot}0357+\log_{10}\left(\frac{k_0}{fN}\right)+\log_{10}\left[{\cdot}216\left(\frac{k(v)}{k_0}\right)+{\cdot}108\left(\frac{k(v+\delta)}{k_0}\right)\right],$$

where

$$v=90{\cdot}36\ \Delta\lambda \quad \text{and} \quad \delta_{D_1}=1{\cdot}898, \qquad \delta_{D_2}=2{\cdot}079.$$

By means of this formula and a corresponding one for $D2$ we can calculate $\log\left(\frac{-\log r}{N}\right)$, and hence derive the theoretical contours for various values of $\log N$.

These were drawn out and measured with a planimeter, and the results gave the following curves of growth for $D1$ and $D2$ at 130° :—

TABLE VII

Equivalent Widths

log N	8	9	10	11
$D1$	·00031	·00241	·0186	·0525
$D2$	·00059	·00446	·0294	·0594

from which the equivalent widths of the observed contours give :

$$D1 : \log N=10{\cdot}51 ; \qquad D2 : \log N=10{\cdot}48.$$

* Mitchell and Zemansky, "Resonance Radiation and Excited Atoms," p. 322, 1934 ; or Unsöld, p. 163.

† Schuler, *Naturwiss.*, **16**, 572, 1928.

The results for other temperatures, although of much smaller weight, may be of interest :

TABLE VIII

T	$D1$		$D2$	
	E.W.	log N	E.W.	log N
120	·0334	10·49	·0384	10·33
110	·0118	9·69	·0218	9·75
102	·0097	9·58	·0183	9·62

We now have to consider how these results will be affected by scattered light in the instrument.

A fraction x of scattered light changes a true residual intensity r to r' where

$$r' = \frac{r+x}{1+x}, \tag{7}$$

and the equivalent width A of a line to A' where

$$A' = \int_{-\infty}^{\infty} (1-r')d\lambda = \frac{1}{1+x}\int_{-\infty}^{\infty} (1-r)d\lambda = \frac{A}{1+x}.$$

Hence for two lines affected by the same amount of scattered light

$$\frac{A_1'}{A_2'} = \frac{A_1}{A_2}. \tag{8}$$

Thus in order to determine the amount of scattered light we increase the equivalent widths of $D1$ and $D2$ in the same ratio until the values of log N agree.

When this operation was carried out on the observed equivalent widths it was found that there could be present not more than 1 per cent. of scattered light, and hence this was neglected.

The true contours taken for the determination of slit pattern were then :

TABLE IX

Residual Intensities

$\Delta\lambda$	$D1$	$D2$	$\Delta\lambda$	$D1$	$D2$
−·08	1·000	0·999	−·01	0·191	0·059
−·06	0·999	0·999	·00	0·055	0·032
−·05	0·999	0·994	·01	0·303	0·097
−·04	0·943	0·843	·02	0·920	0·847
−·03	0·510	0·269	·03	0·998	0·997
−·02	0·241	0·132	·05	1·000	0·999

It was found that these could be treated as if they were infinitely sharp. The determinations from the two lines by Shane's method * agreed exactly, and gave for the slit pattern at slit width 0·025 mm. at the D lines :

* *Lick. Obs. Bull.*, **16**, No. 449, 76, 1932.

TABLE X

$\Delta\lambda$	·00	·05	·10	·15	·20	·25	·30	·35	·40
$K(\lambda)$	4·50	3·54	1·80	1·03	·547	·321	·193	·0965	·0642

where $K(\lambda)$ is normalised.

The curves corresponding to slit widths zero and 0·05 mm. were computed by Miss M. G. Adam. It was now necessary to deduce the slit pattern at *Hα*, and in order to do this it was assumed that since the slit pattern was so broad it must arise mainly from the inhomogeneity of the prisms rather than from diffraction effects. The size of the pattern in mm. on the plate was therefore assumed to be constant, and all that was necessary to change to the new region was to convert the independent variable from mm. to angstroms at the new wave-length, and renormalise the pattern. The error introduced by this procedure will be small for the change from 5890 to 6560.

This gives at *Hα* :

TABLE XI

$\Delta\lambda$	·000	·0702	·140	·211	·281	·351	·421	·491	·561
$K(\lambda)$	3·21	2·52	1·28	·734	·390	·229	·138	·0688	·0457

The advantages of this method of determining the slit pattern have already been discussed in the preliminary account. To recapitulate briefly they are :—

(i) That the slit pattern is determined under conditions which approximate closely to the conditions under which it is to be applied. The exposure times are short and the method easy to operate.

(ii) The calculation of the undistorted contours depends only on a well-tested theory for which all the data can be determined.

(iii) The determination of a pattern to be used on an absorption line is made by the aid of an absorption line. The effect of this is to eliminate errors due to scattering in the emulsion. These are of greatest importance when the instrumental pattern and the line to be measured have nearly the same width.

(iv) A determination of the scattered light in the instrument can be made.

A suggestion due to Dr. Brück which would avoid the necessity of transferring the slit pattern from one spectral region to another is that the resonance lines of other alkali metals should be used to make determinations in other regions.

(5) *The Combination of the Observations.*—It was now necessary to find the best method of combining the individual contours which had been derived. An examination of the tracings showed a large number of lines due mainly to atmospheric water vapour lying in the wings of *Hα* and producing a depression of the shoulders of the line. It was also noticed that the limb plates taken on 1938 November 5 showed much stronger water-vapour lines than the centre plates taken on November 2.

There are two main regions of the spectrum immune from this influence, namely:

(1) 6561·0–6563·8 containing two lines of intensity −2.
(2) 6564·6–6568·5 containing one line −3 and another −2.

An important point to remember is that the continuous background for these lines is the true *Hα* contour at the point, and hence the error in residual intensity produced by them is much reduced if they fall near the centre of *Hα*.

In these two regions the available results were plotted and the values at a number of selected wave-lengths read off. In measuring the tracings all the peaks and valleys at the line centre had been measured in the hope of detecting possible re-emission at the limb. In plotting out the results no smoothing process was carried out and the residual intensities were taken just as they came.

The rest of the curve at the centre of the disc was plotted out by choosing a number of regions at most one or two angstroms in length which contained only faint lines, and for each a point was derived by taking the mean residual intensity and the mean wave-length. It would not have been legitimate to proceed by interpolation, since the line joining an observed point unaffected by water vapour to a point which is so affected only gives the true residual intensity actually at the first point.

Examples of the regions used were:

Table XII

Region	Lines included	No. of Observations
6570·0–6570·5	−3, −2, −3	18
6573–6574	−3	8
6559·8–6560·3	None	19
6558·4–6559·2	−3, −3	24

The same procedure was adopted for the plate at $\cos\theta = \cdot 741$. At the extreme limb a little more care is needed, since when limb plates were taken the water-vapour lines were stronger. A glance at the photographs of the limb tracing (Plate 1) shows that the wings are broken up into a number of peaks by the water-vapour lines. A smooth curve was drawn through the peaks. Some peaks then fell below this curve. The lines on each side of such a depressed point were then measured and, assuming they had the shape of the instrumental pattern, the depression due to water vapour at the peak could be calculated. If this agreed with the depression from the smooth curve at the peak it was assumed that the smooth curve gave the correct wing. In this way those peaks unaffected by water vapour could be selected out. The practice had been adopted of measuring each peak in the wing, and hence for each one of the unaffected peaks a series of measures having a small scatter in wave-length was available. The mean wave-length and the mean residual intensity gave a point on the final curve.

The peaks used had the following approximate wave-lengths :—

6550·0, 6553·2, 6556·5, 6559·0, 6560·0.

In order to fix the values of cos θ for the various strips, tracings were made perpendicular to the solar spectra at 6539·5 (where there is a short region containing no lines) and the values determined by measuring the distance of the strip from the point of inflection of the intensity curve. This was chosen as being the point where the decreasing curve of solar surface intensity joins on to the curve of light scattered on to the sky background just beyond the limb.

Three groups of results having the values of cos θ,

·364, ·303, ·229,

were found.

The nett result was to produce 5 contours of $H\alpha$ at different points on the solar disc derived from about 3000 intensity measures on 56 tracings.

In accordance with the determination from the D lines no correction was made for scattered light.

In order to eliminate the distortion due to the instrument it was necessary to solve the integral equation

$$F(\lambda) = \int_{-\infty}^{\infty} f(\lambda - x)K(x)dx, \qquad (9)$$

where $F(\lambda)$ is the observed contour, $K(x)$ the instrumental pattern given in Table XI and $f(\lambda)$ the true contour which we are seeking.* It was found that the only effect of instrumental distortion was to raise the central intensity by about 4·5 per cent. In one or two cases the observed mean contours showed slight irregularities at the centre. These were not large enough to be significant, and in carrying out the instrumental reduction a smoothed contour was used.

The final results corrected for instrumental distortion are given in Table XIII, while Table XIV gives the cores of the lines before correction.

These tables are self-explanatory except that the line marked "weight" in Table XIII indicates the number of tracings on which the contour is based.

On the whole there is a steady increase in residual intensity at each wave-length as the limb is approached. The general character of the change may be described as follows :—

The change is comparatively slow until about 2/3 of the way from centre to limb except that the shoulders of the line have risen slightly and there has been an increase of 3 to 4 per cent. in the central intensity. The shape of the core of the line is on the whole unchanged. Thereafter both shoulders and central intensity begin to rise more rapidly and the rate of change with cos θ increases. The total width of the line across the wings is not much affected after cos $\theta = ·741$.

* Shane, *loc. cit.*

TABLE XIII

Residual Intensities × 1000

Wave-length	cos θ				
	1·000	·741	·364	·303	·229
6545	995				
48	989				
51	969	979	985	990	995
52	957	969	980	982	991
53	942	959	975	975	985
54	930	950	970	970	980
55	917	941	965	963	977
56	904	930	960	957	975
57	885	915	954	950	971
58	865	898	945	946	966
59	840	876	935	941	959
6560	790	854	917	916	937
61	710	798	867	870	894
61·5	669	750	820	831	859
62	570	655	654	684	702
62·25	458	498	480	502	500
62·5	300	345	317	300	310
62·6	260	275	235	225	260
62·7	205	239	225	213	249
62·8	180	230	224	211	245
62·82	179	230	224	211	245
62·9	185	230	225	213	245
63	242	238	230	224	248
63·25	363	374	362	366	380
63·5	490	506	505	530	528
63·75	586	631	674	706	725
64	630	690	744	759	783
65	750	820	885	889	910
66	817	872	920	929	946
67	853	901	940	950	959
68	880	925	956	959	966
69	896	935	963	965	965
6570	911	945	966	971	964
71	925	951	969	975	966
72	942	959	973	979	971
73	954	965	976	982	976
74	961	971	980	985	981
75	970	978	985	990	987
76	980				
77	984				
78	987				
79	989				
6580	991				
83	996				
E.W.	4·16	3·22	2·42	2·32	2·08
Weight	12	6	10	14	14

TABLE XIV

Uncorrected Cores : Residual Intensities × 1000

Wave-length	cos θ				
	1·000	·741	·364	·303	·229
6562·5	290	345	317	320	332
62·7	243	277	277	274	294
62·75	236	288	284	268	290
62·8	229	295	272	265	296
62·85	230	295	272	268	299
62·9	231	280	273	272	300
63·0	251	281	285	276	301
63·25	353	374	362	366	380

The points most sensitive to experimental error are (i) the centre of the line, where a very large amount of material is needed to overcome the uncertainties due to plate grain; (ii) the steep sides of the core, where the residual intensities are well determined but small errors in wave-length cause a comparatively large effect; and (iii) the extreme wings, where r is greater than ·98 and the influence of plate grain again becomes important. The errors in the first two cases are of the order of 3 per cent. and in the third about 1 per cent. At the shoulders of the line the errors are probably less than this.

The results are illustrated in figs. 6 and 7.

(6) *Comparison with Other Workers.*—The most reliable of the many determinations of the contour of *Hα* at the centre of the solar disc appear to be those of Thackeray * and Minnaert †, while the centre-to-limb variations have been studied by Royds and Narayan.‡ The results may be summarised as follows :—

TABLE XV

Observer	Central Int.	E.W.	Instrument	Plates	Date of Observations
Minnaert	15·0 per cent.	4·04	Grating	Ilford	1927
Thackeray	15·4 ,,	3·54	,,	,,	1934 June
Royds and Narayan	25·1 ,,	2·53	,,	Not stated	1936
Evans	17·9 ,,	4·16	Prism	Ilford	1938 November

In the case of Royds and Narayan no special precautions appear to have been taken to guard against scattered light, and the low value of the equivalent width found by them suggests very forcibly that this effect is present. The qualitative variation of their contours is similar to that found in the present work, but their equivalent widths are everywhere very much lower than ours :

* *M.N.*, 95, 293, 1935.
† *Z. f. Physik.*, 45, 610, 1927.
‡ *Kodaikanal Obs. Bull.*, 109, 1936.

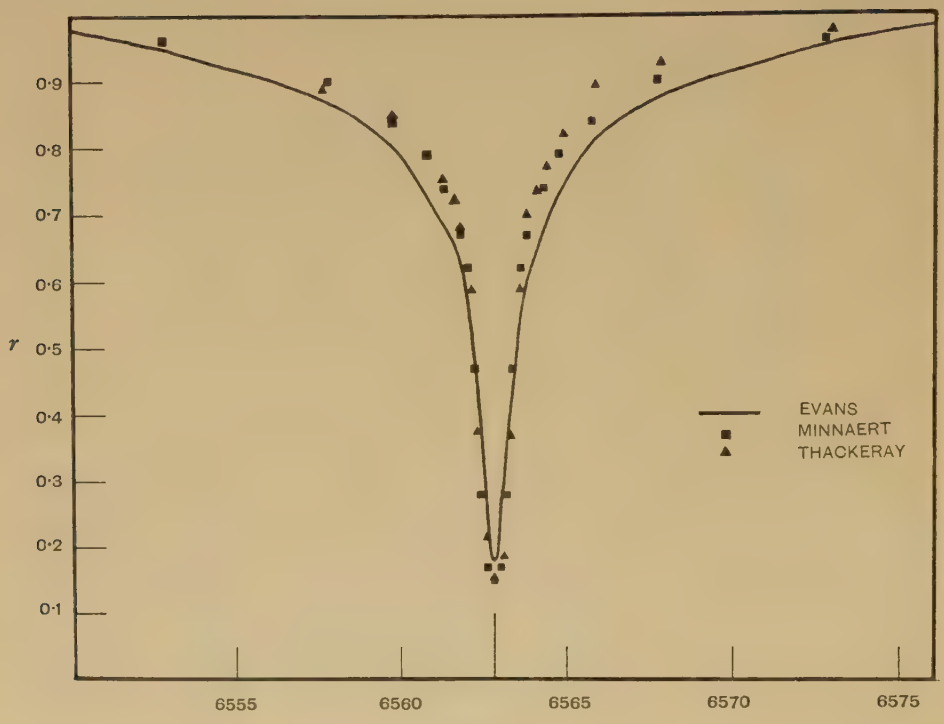

0·9
0·8
0·7
0·6
r
0·5
0·4
0·3
0·2
0·1
6555
6560
6565
6570
6575
EVANS
MINNAERT
THACKERAY

FIG. 6.

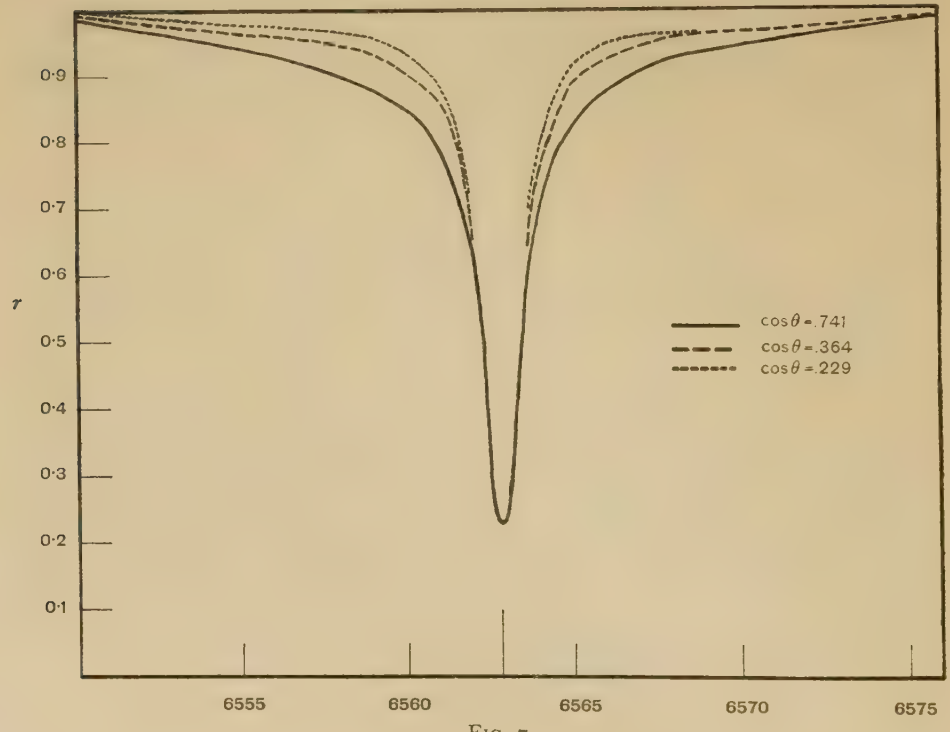

0·9
0·8
0·7
0·6
r
0·5
0·4
0·3
0·2
0·1
6555
6560
6565
6570
6575
cos θ = .741
cos θ = .364
cos θ = .229

FIG. 7.

TABLE XVI

Equivalent Widths

Observer	cos θ				
	1·000	·741	·364	·303	·229
R. & N.	2·53	2·30	1·63	1·52	1·39
Evans	4·16	3·22	2·42	2·32	2·08

Turning to the work of the other observers we find a much more strictly controlled set of conditions. Minnaert determined the intensities of the ghosts of his grating and used filters to cut down the range of wave-lengths admitted to the instrument. Thackeray used a monochromator and, in order to relieve his plates of the strain of registering the whole intensity range from wing to centre in one step, he determined the centre from dense plates, using the monochromator and the wings from lighter plates without the monochromator, and fitted the two portions together. The results are compared in Table XVII.

TABLE XVII

Residual Intensities × 1000

Δλ	Observer				$M-E$	T_c-E
	Minnaert	Thackeray	Thackeray (corr.)	Evans		
−10	960	930	917	942	018	−025
− 5	900	890	870	870	030	000
− 3	840	854	826	800	040	026
− 2	790	789	749	730	060	019
− 1·5	740	753	708	685	055	023
− 1·0	670	681	623	618	052	005
− 0·5	375	374	...	425	−050	...
− 0·2	170	200	...	265	−095	...
0·0	150	154	...	179	−029	...
0·2	170	180	...	234	−064	...
0·5	375	367	...	391	−016	...
1·0	670	700	644	597	073	047
1·5	740	771	732	671	069	061
2·0	790	820	787	730	060	057
3·0	840	894	874	807	033	067
5·0	900	924	909	875	025	034
10·0	960	972	966	954	006	012

The differences (which are illustrated in fig. 6) are large. In effect the core of the line as measured by us is narrower than that given by the other two observers, and the shoulders as measured by us are lower than those given previously. Thackeray's values are considerably higher than Minnaert's on the right-hand shoulder and in fair agreement on the left (shorter wave-length) side.

First let us consider Thackeray's results. He states that the wings of the line were determined without the monochromator, and also that without the monochromator he found a central intensity of 28·2 per cent.

Applying equation (7) we find that this corresponds to no less than 17·8 per cent. of scattered light, and that therefore for all the region determined without the monochromator this amount of scattered light must be present.

The column "Thackeray (corr.)" in Table XVI gives the values when the correction for this is applied. In the blue wing this brings Thackeray's results almost down to ours and halves the discrepancy in the red wing. The change brings Thackeray into agreement with Minnaert in the red wing.

The remaining differences (listed in columns $M-E$ and T_c-E in Table XVII) can only be due to

(i) Errors in characteristic curves.
(ii) Errors in continuous backgrounds.
(iii) Real changes in the contours.

If our unorthodox method of drawing characteristic curves has introduced errors into our results they will reveal themselves as a correlation between determined residual intensity and plate opacity. No such correlation could be found for plates ranging in opacity from 0·40 to 0·97.

From Table XV we see that all three observers used Ilford plates, and hence, although no remark to this effect is to be found in the papers of either Thackeray or Minnaert, both must have been faced with the problem of the rapidly varying plate sensitivity. There can be no error in our results from this cause because of our use of artificial backgrounds. Without knowing the densities of Thackeray's and Minnaert's plates it is difficult to estimate the error which could be introduced in this way, but we can say that the effect will be a positive error, larger in the red wing than in the blue, whose amount may easily be 4 per cent. or even more. This is sufficient to bring the differences within the range of experimental error, and there seems to be no evidence for postulating a real change in the contour of the line.

With regard to the central intensity it seems clear that a very large amount of material is necessary to overcome the uncertainties due to plate-grain, and that our policy of combining the contours without smoothing may well have led to a central intensity which is less reliable than Minnaert's.

(7) *Theoretical Discussion.*—For the discussion of the results we use equation (1). For η, ϵ and δ constant we find

$$r(\theta)=\frac{q^2}{s^2}\frac{1}{1+b/a\cos\theta}\left\{1+\frac{b/a\cos\theta}{1+\eta}+\frac{\eta(1-\delta)(b/a-1{\cdot}5\sqrt{1+\eta})}{(1+\eta+s\cos\theta)(s+1{\cdot}5\sqrt{1+\eta})}\right\} \quad (10)$$

where

$$q^2=3(1+\eta)(1+\epsilon\eta),$$
$$s^2=3(1+\eta)(1+\epsilon\eta+\delta\eta),$$

and we have taken

$$\mathcal{J}_T=a+b\tau.$$

Consider the points $\cos\theta = 1{\cdot}000$ and $\cos\theta = {\cdot}303$. Then from the observations of Moll, Burger and van der Bilt * we find $b/a = 1{\cdot}10$. Consider the function

$$f(\cos\theta) = r(\theta)\left(1 + \frac{b}{a}\cos\theta\right), \tag{11}$$

which can be calculated from the observations at each wave-length, while the right-hand side can be computed as a function of η, ϵ and δ from equation (10).

When $\epsilon = 0$ and $\delta = 1$, we find

$$f(\cos\theta) = \frac{1}{1+\eta}\left\{1 + \frac{b/a\cos\theta}{1+\eta}\right\}, \tag{12}$$

and hence the line does not vanish at the limb. For discussion we take the wave-lengths in column 1 of Table XVIII. The second column gives the values of $f(\cos\theta)$ computed from the observations, while the other columns give values of $\rho\left(=\frac{1}{1+\eta}\right)$ for the values of δ listed (assuming $\epsilon = 0$).

These may be found graphically from fig. 8.

TABLE XVIII

Values of ρ

Wave-length	$f(\cos\theta)$		δ			
			0·0	0·2	0·5	1·0
6561·5	Centre	1·401	·450	·580	·680	·761
	Limb	1·108	·600	·720	·803	·860
6562·8	Centre	·376	·022	·120	·197	·287
	Limb	·281	·030	·112	·180	·260
6567·0	Centre	1·793	·749	·808	·860	·905
	Limb	1·266	·870	·920	·945	·960
6570·0	Centre	1·917	·848	·885	·917	·945
	Limb	1·297	·925	·959	·970	·976

From this table ρ, δ curves may be plotted at each wave-length for centre and limb, and their intersection gives the values of ρ and δ there. Except at 6562·8 there is no solution. Always $\rho_l > \rho_c$ indicating that whatever the value of δ, η increases with depth in the atmosphere. At 6562·8 we find $\delta = 0{\cdot}10$. A solution is presumably possible here because the penetration of the line of sight into the atmosphere is small both at centre and limb, and so η has no chance to change enough to prevent a solution being found. However, the value found is not well determined, since the ρ, δ curves at centre and limb cut at a very acute angle.

The observations can be analysed in another way. For each wave-

* *B.A.N.*, **91**, 83, 1925.

length at the centre of the disc we can find ρ for $\delta = 0$ and $\delta = 1$ from the observed residual intensities; using these values of ρ we can in each case compute $f(\cdot 303)$, and by comparing the computed values with the observed values we can see which value of δ gives the better fit. ϵ is taken to be zero as before.

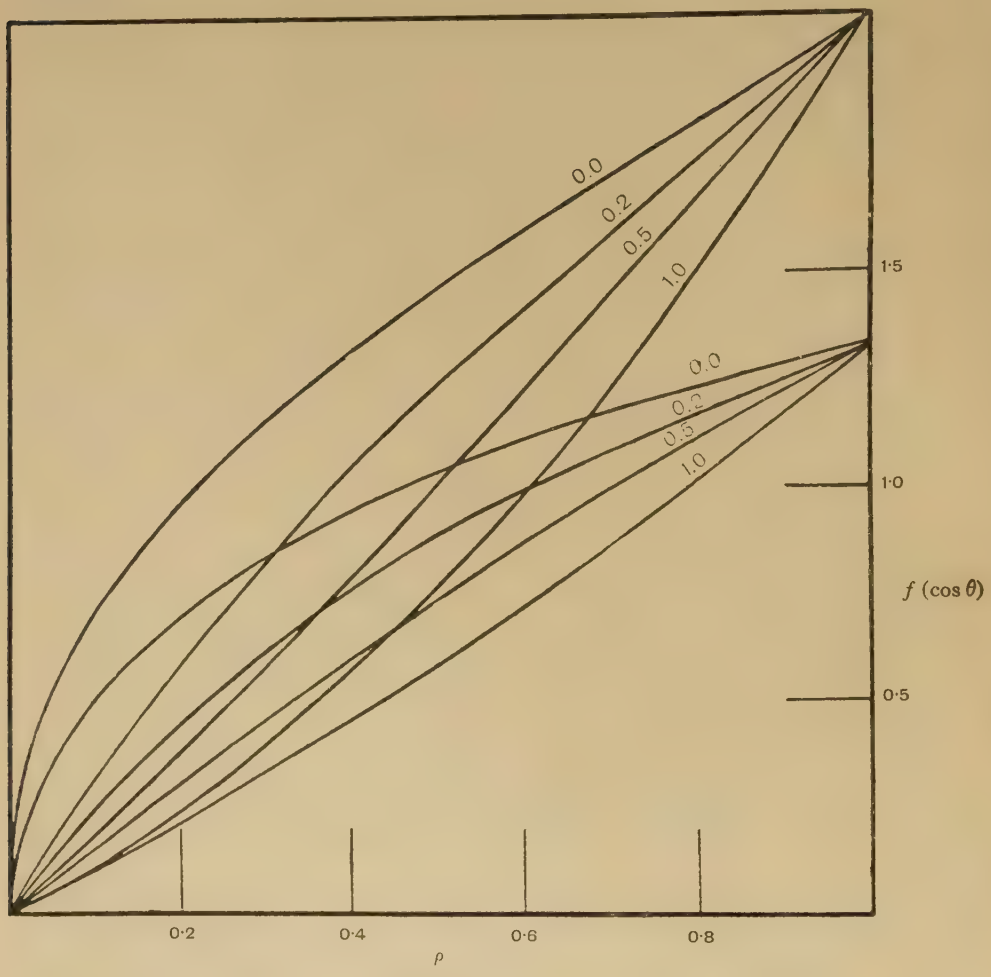


Fig. 8.

We find:

Table XIX

Wave-length	$f(\cdot 303)$ (Obs.)	$f(\cdot 303)$ (calculated)	
		$\delta = 0$	$\delta = 1$
61·5	1·108	1·000	0·960
62·8	0·281	0·240	0·320
67·0	1·266	1·21	1·18
70·0	1·297	1·25	1·24

There is thus a slight improvement for $\delta = 0$ over $\delta = 1$.

We can now make the following deductions :—

(i) η increases with depth at each wave-length.
(ii) There is some slight evidence that δ is small.
(iii) The figures given in Table XIX show that δ is not well determined. The function on the right-hand side of equation (10) possesses the peculiar property that the residual intensity at the limb is determined almost entirely by the intensity at the centre of the disc, and is almost independent of the choice of ρ and δ which give that intensity. This holds over the large range of values of ρ listed in Table XVIII.

Summary

The contour of *Hα* has been measured at the centre and limb of the Sun. New methods using the D lines of sodium in absorption have been developed for the determination of the instrumental pattern and of the amount of scattered light. A new method of drawing characteristic curves has been used, and the continuous spectrum background has been estimated experimentally.

The residual intensity has been found to increase at each wave-length in the line as the limb is approached.

Unsöld's suggestion that the Balmer series is formed effectively by pure absorption is criticised and a new equation of transfer proposed. Some evidence is found in support of the view that uncompensated cycles are not of importance in the formation of *Hα*.

The author wishes to take this opportunity of expressing his gratitude for the unfailing advice and guidance of Professor Plaskett throughout this work. He also wishes to thank Miss M. G. Adam for many valuable suggestions and Mr. G. Neale for his cordial co-operation in the design and construction of apparatus.

University Observatory,
Oxford :
1940 *January.*

A SENSITOMETRIC STUDY OF SOME DEVELOPERS AND EMULSIONS OF ASTROPHYSICAL INTEREST

D. R. Barber

1. *Introductory.*—It is a recognised fact that the effective response of a photographic emulsion to light stimuli, measured in terms of the density of the developed image, will depend, among other factors, upon the nature of the developing agent.

The present paper deals with the results of an investigation into the effect of three different developers when used in conjunction with a range of negative materials, comprising three blue sensitive, three "orthochromatic" and three panchromatic emulsions.

From the astrophysical standpoint the ideal developer should possess a variety of diverse characteristics. High contrast, combined with a low value of background (chemical) fog, is an important desideratum. In addition, freedom from excessive graininess in the case of super-speed materials is a valuable asset. In practice, a compromise must suffice since it is inevitable that any pronounced reduction of graininess which may result from the use of ultra-fine grain emulsions, or specially compounded developers, will be gained only at the expense of a lowered effective threshold response. For this reason no ultra-fine grain developer was included amongst the types selected for the present investigation.

2. *Experimental.*—The various brands of plates which were used are listed in Table I, and are there arranged in three groups in order of their effective threshold response—a criterion of some importance to astrophysical workers. They were chosen to cover a wide range of effective speed, spectral response and contrast.

No. 1—a relatively slow emulsion—has characteristics very similar to those of the "process" variety, and is included on account of its frequent use in the production of diapositives, as well as in solar photography.

Nos. 2 and 3 are both high-speed materials, supersensitive to the photographic spectral region.

Nos. 4, 5 and 6 possess, in addition, green and yellow response, their sensitivity to the blue, violet and ultra-violet being of the same order as No. 3.

In the panchromatic group No. 7 is a fast emulsion, specially sensitised for colour-temperature investigations, and has a spectral response extending to 7110 A. It is, however, slower than the hypersensitive emulsions, Nos. 8 and 9, which possess very high speeds in both blue and red regions of the spectrum.

In order to obtain a direct comparison of the influence of the nature of the developer on any one type of emulsion, a plain metol-hydroquinone

TABLE I

Emulsion	Sensitometric Data: Exposure Time (secs.) Blue	Red	Relative Transmission * Blue	Red	Development Time (mins. at 65° F.) M.Q. M.C.	R.
Blue Sensitive :						
1. Ilford Ordinary	30	...	28·5	...	3	4
2. Ilford Zenith	180	...	1·0	...	6½	10
3. Barnet Super Press	120	...	1·0	...	8½	13½
Orthochromatic :						
4. Imperial Eclipse O.	120	...	1·0	...	8½	13½
5. Ilford Iso-Zenith	180	...	1·0	...	8½	13½
6. Ilford Double X-Press	90	...	1·0	...	6	9
Panchromatic :						
7. Ilford W. 8917 P.	180	180	1·0	1·5	6½	10
8. Ilford H.S.P.	180	180	1·0	1·5	6½	10
9. Ilford H.P. 2	180	180	1·0	1·5	6½	10

* Colour filter plus absorbing and diffusing screens.

developer having the following formula was adopted as the standard of reference :—

Standard Developer (M.Q.)

Metol	70 grains
Hydroquinone	240 ,,
Sodium sulphite	7 ounces
Sodium carbonate	6 ,,
Potassium bromide	60 grains
Water to make	80 ounces

(For use take 1 part of stock solution to 1 part of water.)

Of the two other developers used in the investigation, one was a proprietary brand, "Tabloid Rytol" Universal Developer (R), which had previously been used at this observatory over a number of years.

The other was a two-solution developer of the free-alkali type, as used in the process trade for the production of high contrast on slow-speed material. It was compounded according to the following formula :—

Maximum Contrast Developer (M.C.)

(*a*)	Sodium bisulphite	110 grains
	Hydroquinone	110 ,,
	Potassium bromide	110 ,,
	Water to make	10 ounces
(*b*)	Sodium hydroxide	200 grains
	Water to make	10 ounces

(For use mix equal quantities of (*a*) and (*b*).)

All the plates used in this research had been previously stored in a refrigerator at 35°–40° F., and, after exposure in the sensitometer, were developed at 65° F., the temperature of the developing bath being maintained constant to within ± 1° F. by manual control of heater lamps placed beneath the developing tray. All plates were handled in total darkness prior to development, which was carried out in a covered dish. The times of development for each emulsion and developer are given in Table I.

The tube sensitometer used for calibrating each plate has been described previously.* Since the effective response of a photographic material is known to depend upon the quality of the radiation incident upon it, all plates were calibrated with blue light through a Wratten No. 47*a* cemented filter, having a transmission band from 3750 to 5200 A. In the case of emulsions Nos. 7, 8 and 9 an additional calibration was made through a Wratten No. 71*a* filter passing red light from 6000 A. up to the limit of the spectral response of the emulsion.

The intensity level of illumination at the six tube apertures was adjusted to an appropriate value by inserting silver absorbing-screens between the opal glass diffusers placed below the colour filter in the sensitometer. The densities of the developed "spot" images were thereby kept within the range of accurate measurement by the microphotometer.

Exposure times were equalised, as far as was possible, so as to avoid a time ratio in excess of 1 : 6. Further, in order to eliminate the effect of reciprocity failure when comparing the influence of different developing agents on the same emulsion, each of the test-plates received the same intensity scale, and time, of exposure.

The relative transmissions of the various combinations of filter, absorbing screen and diffuser, together with the exposure times, are listed in Table I.

The complete series of standardised plates, twenty-seven in all, was measured in the "Oxford" microphotometer †, using a projected slit width of 31·3 μ. Observations of galvanometer deflection were made visually from the glass scale. In order to compensate for small changes in glass transmission from one plate to another, the "clear glass" reading was adjusted in each case to correspond to full-scale (30 cm.) deflection. All density measures were made against this "clear glass" reading. The values of development (chemical) fog obtained for each combination of emulsion and developer tested are recorded in Table III, and they represent the mean of six measures obtained from different regions of the plate background.

Measures of "spot" density which form the basis of the characteristic curve were derived from a mean of numerous readings taken while the analysing beam slowly traversed each spot.

In addition, a series of tracings at a constant density level of 0·5 was made for each emulsion used in conjunction with the different developers, photographic contact copies of a continuous wedge being used for this purpose. The tracings enable a direct comparison to be made of the graininess of the developed images.

* *N.L.O. Bull.*, **1**, 14, 1938.
† *M.N.*, **95**, 160, 1934 ; *N.L.O. Bull.*, **2**, 20, 1939.

A suggestion was made that the numerical factors used as a basis for the well-known Watkins' system of development to a predetermined contrast level might be employed as a useful means of developer classification. Accordingly, these factors were determined experimentally for four representative emulsions developed in each of the three reagents at 65° F. For this purpose an additional series of plates was first exposed to blue light in the sensitometer, and then processed, using an appropriate safe-light. The factors were computed for the same total times of development (and contrast values) as were used for the main series of test-plates.

As will be seen from Table II, the "Watkins' factor," W, is found to vary markedly with the character of the emulsion, in addition to its known dependence upon the contrast factor, γ.

TABLE II

Emulsion	M.Q.		M.C.		R.	
	γ	W	γ	W	γ	W
1. Ilford Ordinary	2·5	10½	4·6	9½	1·6	11½
3. Barnet Super Press	2·0	18	3·0	15	1·6	19
6. Ilford Double X-Press	2·9	14	3·0	10	1·7	18
8. Ilford H.S.P.	2·3	11	2·5	8	1·2	11

In consequence, the "Watkins' factor," as normally employed, is not a reliable index of the behaviour of different developing reagents, when used with a variety of emulsions whose characteristics differ widely.

3. *Discussion of the Results.*—The characteristic curves derived from the microphotometer measures were used to evaluate contrast factor and sensitivity, for each of the emulsions tested in combination with the three developers. They were obtained by plotting the relation between fog-free density (D) and $\log_{10}(It)$, where (D) is defined as $\log_{10}(S_0/S) - S_0$ and S being the respective galvanometer scale deflections corresponding to "clear glass" and "spot," and (It) is the relative exposure given in the sensitometer.

A study of these curves has revealed a number of interesting facts. Throughout the series it is found that the linear portion of the characteristic for plates developed in the standard M.Q. occupies a position intermediate between those for plates processed with M.C. and "Rytol" developers. For emulsions Nos. 4, 5 and 9, the three characteristic curves intersect one another in the region of the "toe." As might be expected, the effect of using a free-alkali developer is, with one exception (emulsion No. 5—10 per cent. decrease), materially to increase the contrast factor; and for the panchromatic materials this effect is greatest for red light. Maximum and minimum percentage increases are 84 and 3 respectively for emulsions Nos. 1 and 6. The use of "Rytol," however, is found to lower contrast markedly, the percentage decrease in γ varying from 10 per cent. in the case of emulsion No. 9 to nearly 50 per cent. for emulsion No. 8. Values of the contrast factor, γ, are tabulated for reference in Table III.

TABLE III

Emulsion	Development Fog (D_f)			Contrast Factor (γ)					
				Blue Light			Red Light		
	M.Q.	M.C.	R.	M.Q.	M.C.	R.	M.Q.	M.C.	R.
Blue Sensitive :									
1. Ilford Ordinary	0·01	0·04	0·01	2·5	4·6	1·6	...	...	...
2. Ilford Zenith	0·13	0·13	0·13	1·8	2·1	1·6	...	...	...
3. Barnet Super Press	0·44	0·73	0·43	2·0	3·0	1·6	...	...	...
Orthochromatic :									
4. Imperial Eclipse O.	0·19	0·25	0·87	2·4	2·6	1·9	...	...	...
5. Ilford Iso-Zenith	0·17	0·33	0·94	2·9	2·6	1·7	...	...	...
6. Ilford Double X-Press	0·24	0·54	0·27	2·9	3·0	1·7	...	...	...
Panchromatic :									
7. Ilford W. 8917 P.	0·19	0·46	0·35	2·3	2·6	1·9	2·7	3·7	2·3
8. Ilford H.S.P.	0·14	0·27	0·33	2·3	2·5	1·2	1·9	2·8	1·2
9. Ilford H.P. 2	0·32	0·55	0·37	2·0	2·9	1·8	2·1	3·9	1·9

The influence of the developer upon the formation of chemical fog is seen to vary with the nature of the emulsion, the results being also tabulated in Table III. In the first group it is found that free alkali in the developer results, in nearly every instance, in raising the background fog level. On the other hand, "Rytol" is found to yield results similar to those obtained by using the standard M.Q. developer. For ortho- and panchro- materials, both the developing agents tested against the standard suffer from the disadvantage of producing a high level of fog density. This is notably the case for emulsions Nos. 4 and 5 when developed in "Rytol," the level being raised to approximately five times the standard. When it is remembered that the total density of silver deposit limits the degree of accuracy with which any plate may be analysed by the microphotometer, it will be apparent that an excessive production of background is likely to annul any advantages which may accrue from the use of such developers, as regards increased contrast and effective speed.

A factor which is frequently of more importance in the field of astrophotography than the gamma value of the emulsion is its effective sensitivity. The more usual definition of this function is derived from the well-known equation of Hurter and Driffield,

$$D = \gamma(\log_{10} E - \log_{10} i),$$

where D = measured density of silver deposit.

E = exposure required to produce a density D.

γ = the contrast factor, as measured by the slope of the linear portion of the characteristic curve.

i = the "inertia" value, defined as the value of E measured at the point of intersection of the prolongation of the linear part of the characteristic curve on the log E axis.

The "effective speed," or sensitivity of the emulsion, is then expressed in terms of an arbitrary parameter (k/i), and is termed the H and D number.

Such a criterion will have little, or no, significance in many branches of astronomical photography. As Th. Dunham, jun.*, has pointed out, a much better estimate of the effective response of an emulsion under the conditions of exposure associated with astrophysical investigations will be that based upon an evaluation of the exposure required to produce that minimum density which is adequate for the particular work in view. Thus, in the examination of direct star fields, clusters, etc., the value of the effective threshold response † will be the criterion adopted; whilst for spectrophotometric exposures a minimum density level of 0·5–0·6 appears to be the threshold value necessary to ensure the detection and accurate measurement of faint spectral lines or bands.

Accordingly, in Table IV there will be found two separate values of effective sensitivity (S), viz. $S_{0\cdot05}$, which is a simple function of the exposure necessary to produce a "threshold" silver deposit of density 0·05 above fog level, and $S_{0\cdot5}$, which represents the sensitivity at the higher density of 0·5.

TABLE IV

Emulsion	Percentage Increase (+) or Decrease (−) in Effective Sensitivity, $S=10^4/E$							
	$S_{0\cdot05}$				$S_{0\cdot50}$			
	Blue		Red		Blue		Red	
	M.C.	R.	M.C.	R.	M.C.	R.	M.C.	R.
Blue Sensitive :								
1. Ilford Ordinary	+26	−20	...	...	+49	−18	...	...
2. Ilford Zenith	0	−16	...	...	+18	−57	...	...
3. Barnet Super Press	+26	−60	...	...	+32	−63	...	...
Orthochromatic :								
4. Imperial Eclipse O.	0	+ 5	...	...	+18	−57	...	...
5. Ilford Iso-Zenith	−35	−65	...	...	0	−72	...	...
6. Ilford Double X-Press	+58	−15	...	...	+70	−29	...	...
Panchromatic :								
7. Ilford W. 8917 P.	0	−40	+37	−28	+15	−26	+12	−21
8. Ilford H.S.P.	+26	0	0	−29	+18	−31	+15	−47
9. Ilford H.P. 2	−20	−41	+26	−45	+32	−17	+27	−24

For convenience of presentation the sensitivity values at the two density levels are expressed as the percentage increase, or decrease, referred to

* *Die Anwendung der Photographie in der Astrophysik*, p. 287.

† The continental Scheiner system of speed classification is based upon that exposure necessary to give the minimum measurable density when the sensitive material is subjected to standard conditions of exposure and development. It is therefore a measure of the actual threshold sensitivity of the emulsion. In the similar DIN system the response is measured at a density level of 0·1 above background fog.

standard M.Q., obtained by development in M.C. developer and in "Rytol." Some interesting facts emerge from a study of these figures.

It will be noted, for example, that in one instance only (emulsion No. 4) does the use of "Rytol" raise the threshold response; whilst at the 0·5 level all the emulsions tested show a marked decrease in effective sensitivity when processed in this developer. In contrast to this, plates developed in the caustic hydroquinone reagent gain appreciably in threshold speed in six instances, a notable increase of nearly 60 per cent. being recorded for emulsion No. 6. In two instances a lowered threshold value is found, viz. for Nos. 5 and 9. At the higher density ($D = 0{\cdot}5$) the gain in sensitivity is general, only one emulsion (No. 5) failing to respond. As at the threshold, the largest gain is shown by emulsion No. 6.

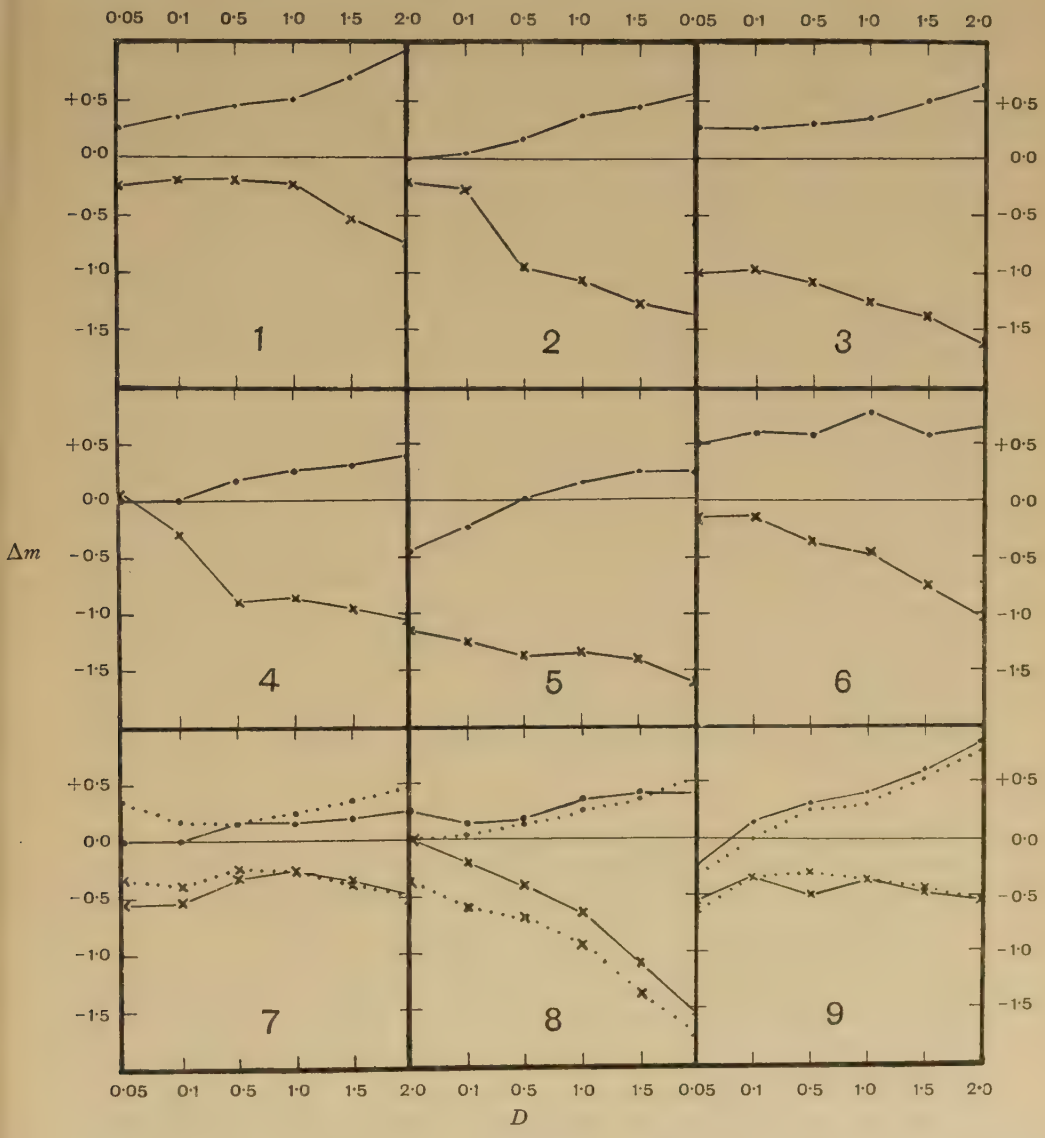
In a consideration of the above results it is necessary to take into account all the concomitant factors. Although it has been shown that the use of the free alkali-type developer raises the threshold, it has to be remembered that this advantage is obtained, in the majority of instances, only at the expense of a disproportionate increase of chemical fog and graininess. In this connection F. E. Ross * has found that such developers are unsuited to the development of negatives intended for the measurement of stellar diameters or positions, or for the precise evaluation of the positions of spectral lines. This is on account of the distortion of the images, which results from a strong contraction of the gelatine film on drying. A similar contraction is noted whenever a developer of the pyro-metol type is employed. Some experiments conducted at this observatory, in which a direct comparison of astrographic negatives developed in M.C. and M.Q. developers was made, have confirmed the above statement.

A more detailed study of the response of each emulsion-developer combination is summarised graphically in the diagram. Here the observed change in sensitivity is recorded in terms of the magnitude gain or loss which results from the use of either M.C. or "Rytol" developers in place of the standard M.Q. It is considered that this form of graphical presentation will be more useful to those engaged in astronomical photography.

For seven of the emulsions tested it will be observed that a development in "Rytol" results in a progressive loss in magnitudes as the density level is raised above the background fog. For the panchromatic emulsions Nos. 7 and 9, the magnitude loss is nearly constant throughout the density range, and it amounts to approximately 0·5 mag. In one instance only, viz. for emulsion No. 4, a slight threshold gain is produced by this developer.

The graphs for the three panchromatic materials exhibit significant differences in their response to blue and to red light. The effect is greatest for emulsion No. 7, for which it will be noted that the "blue" and "red" curves for the M.C. developer have opposite curvature. The maximum threshold gain (for blue light) is obtained by the combination of emulsion No. 6 and M.C., this increase amounting to 0·5 mag. Two emulsions, Nos. 5 and 9, show a loss of speed at the threshold when processed in this developer.

* *Physics of the Developed Photographic Image*, p. 173.



Magnitude Gain or Loss for various Emulsions processed with M.C. or "Rytol," referred to M.Q.

1. *Ilford Ordinary.* 2. *Ilford Zenith.* 3. *Barnet Super Press.*
4. *Imperial Eclipse O.* 5. *Ilford Iso-Zenith.* 6. *Ilford Double X-Press.*
7. *Ilford W.* 8917 *P.* 8. *Ilford H.S.P.* 9. *Ilford H.P.* 2.

M.C. —·—·—·— *Blue* ... . *Red* *R.* —×—×— *Blue.* ...×.....×... *Red.*

The effect of each emulsion-developer combination upon the "grain" of the silver image has been studied by means of a comparison of the series of microphotometer tracings already referred to. The amplitude of the tracing was used as a criterion of graininess, and it was found that there is, as may be expected, a pronounced increase in "grain" in passing from the

slow-speed emulsion No. 1 to the faster ortho- materials Nos. 4, 5 and 6. The panchromatic emulsions Nos. 7, 8 and 9 show somewhat less grain than those of the ortho- variety. The effect of using the caustic hydroquinone developer is to increase graininess for all of the emulsions tested. Plates processed with "Rytol" also show a rather large increase for the majority of emulsions. Differences existing between panchromatic plates calibrated with blue and red light are slight for all three developers.

The main conclusion reached from a consideration of the results of the investigation is that the standard metol-hydroquinone developer, even when used with a wide variety of sensitive materials, invariably produces negatives possessing to a high degree those qualities which are important from the astrophysical point of view.

In special circumstances the property of raising the threshold density possessed by the free alkali (M.C.) developing agent, when it is used in conjunction with a particular emulsion, *e.g.* No. 6, may be of some importance.

The lowered contrast, and effective sensitivity, resulting from the use of "Rytol" developer, under normal time and temperature conditions, lead to a necessary modification of processing technique, the time being often unduly prolonged in order to secure adequate density. This, in turn, is productive of high values of background fog, and thus restricts the general use of the developer. But, on account of the valuable properties of non-deterioration on prolonged storage, and energetic action at low temperatures which "Rytol" possesses, its use is occasionally indicated.

Summary

1. An investigation is described in which the influence of three different developers upon nine dissimilar emulsions is examined from an astrophysical standpoint.

2. Values of background (chemical) fog, contrast factor and effective sensitivity, at different density levels, have been derived from microphotometer measures of plates exposed in a tube sensitometer. The influence of the developing agent upon the quantities enumerated is discussed for the various emulsions tested.

3. A series of diagrams is reproduced to illustrate graphically the change in the effective response of the emulsion at different densities, expressed as a magnitude gain, or loss, referred to standard development.

4. Comparisons are made of the graininess of the developed image for each of the emulsion-developer combinations. These have been derived from microphotometer tracings at a constant density level of 0·5.

5. It is concluded that the standard metol-hydroquinone developer is the most suitable for use in astronomical photography. It is capable of yielding negatives of excellent quality when used in combination with a wide range of emulsion types.

Norman Lockyer Observatory,
Sidmouth, Devon :
1939 *October* 16.

RELATIVE GRADIENTS OF 250 STARS DETERMINED AT THE ROYAL OBSERVATORY, GREENWICH

(Communicated by the Astronomer Royal)

The present catalogue of spectrophotometric gradients of 250 stars continues the results given in *Observations of Colour Temperatures of Stars, 1926–1932*, and represents the completion of the Greenwich colour temperature programme. The list includes most northern stars brighter than $4^{m}\cdot5$ whose H.D. spectral types are Oe5, B or A, as well as many fainter ones of these types and a selection of bright F- and G-type stars. Each star has been compared at least four times with one or more of the Greenwich standard stars, and many of them oftener. The average probable error of the listed gradients may be taken as $\pm$ ·02 for the catalogue stars, and $\pm$ ·013 for the standard stars, which are printed in heavy type in the list.

The methods used in determining the gradients are not essentially different from those described in the volume referred to above. The main changes are the replacement of the 30-inch reflecting telescope by the new 36-inch Yapp reflector ; the application (in the later work on fainter stars) of small corrections to allow for the difference in exposure time between the stellar exposure and the calibration exposure ; and the use of the more sensitive Ilford "Astra VIII" plates instead of the plates previously employed. Full details of the work will be given in a forthcoming Greenwich volume, publication of which is at present held up on account of the war. Meanwhile it is safe to assume that the gradients given below are uniform with those of the 1932 volume, any small differences between the two lists being due to the incorporation of additional material.

Relative gradients only are given, since a definite value of the quantity needed to convert these to absolute gradients is not yet available. The zero to which these are referred is the same as before, namely, the mean of nine specified stars of H.D. type Ao. Absolute values, and hence "temperatures" (see Table V, p. 63, *Observations of Colour Temperatures of Stars*), can be subsequently obtained by adding whatever value is ultimately found best for the Greenwich zero-point ; provisionally it may perhaps be put at about +1·10, corresponding to 15,600° K. The average Ao star is now found, however, to have a small positive gradient on this system, especially if Mt. Wilson types are taken ; on this latter basis about 12,700° K. is the "temperature of an Ao star", but on any system the scatter appears considerable.

Each star in the list is identified by a serial number, its name, its R.A. and declination for 1950·0, and its visual magnitude and spectral type from the H.D. catalogue ; except that where the two components of a double star were known to be separated on the Greenwich plates, the magnitude is that of the component measured. In some cases even if the spectra

overlapped it was still possible to measure in an undisturbed strip. Where the spectra are known to be superimposed, the combined magnitude is given whether or not any trace of the secondary spectrum could be detected ; this applies of course in particular to all spectroscopic binaries. In col. 7 appears the letter *c* in all cases where Mt. Wilson Contribution No. 511 or 512 indicates that the spectrum shows supergiant characteristics, and the letter *e* where Merrill and Burwell's catalogue (*Ap. J.*, **78**, 87, 1933) indicates the presence of emission lines. Cols. 8 and 9 give the approximate galactic co-ordinates taken from *Ann. Obs. Lund*, **3** (1932). The final mean gradient appears in the last column ; the values in heavy type refer to the Greenwich standard stars.

The observations, first with the 30-inch reflector and later with the 36-inch reflector, were mainly shared by Messrs. Greaves, Davidson, Martin and Chamberlain, Dr. Atkinson taking the place of Mr. Greaves in the final stages.

The measurement of the plates in the photometer was carried out at first by Messrs. Greaves, Davidson and Martin. Subsequently Mr. Martin took over the responsibility of this important part of the programme, assisted by Miss Cumberledge.

No.	Star	R.A. 1950·0	Dec. 1950·0	Vis. Mag.	Spectrum H.D.	Remarks	Galactic Long.	Galactic Lat.	Gradient *G*
		h m	° ′				°	°	
1	α Andr	0 5·8	+28 49	2·15	A0*p*		80	−33	**−0·10**
2	β Cass	6·5	58 52	2·42	F5		85	− 3	**+0·69**
3	γ Pegs	10·6	14 54	2·87	B2		79	−46	**−0·30**
4	θ Andr	14·5	38 24	4·44	A2		84	−24	**+0·14**
5	σ Andr	15·7	36 30	4·51	A2		84	−25	**+0·13**
6	κ Cass	30·1	62 39	4·24	B0	*e c*	88	0	**+0·45**
7	ζ Cass	34·2	53 37	3·72	B3		89	− 9	**−0·26**
8	π Andr	34·2	33 27	4·44	B3		88	−29	**−0·04**
9	ξ Cass	39·3	50 14	4·85	B3		89	−11	**−0·04**
10	o Cass	41·9	48 1	4·70	B2	*e*	90	−14	**−0·02**
11	ν Andr	47·0	40 48	4·42	B3		91	−21	**−0·15**
12	γ Cass	53·7	60 27	Var.	B0	*e*	91	− 2	Var.
13	ϕ Andr	1 6·6	46 59	4·28	B8		94	−15	**+0·06**
14	δ Cass	22·5	59 59	2·80	A5		95	− 2	**+0·25**
15	υ Andr	33·9	41 9	4·18	G0		100	−20	**+1·00**
16	ϕ Pers	40·5	50 26	4·19	B0	*e*	99	−11	**+0·22**
17	α U Min	48·8	89 2	2·12	F8	*c*	90	+27	**+1·01**
18	α Tria	50·2	29 20	3·58	F5		107	−31	**+0·99**
19	ϵ Cass	50·8	63 25	3·44	B3		97	+ 2	**−0·20**
20	β Arie	51·9	20 34	2·72	A5		111	−39	**+0·29**
21	β Tria	2 6·6	34 45	3·08	A5		109	−24	**+0·29**
22	γ Tria	14·3	33 37	4·07	A0		111	−25	**+0·05**
23	41 Arie	47·0	27 3	3·68	B8		121	−28	**−0·08**
24	γ Pers	3 1·2	53 19	3·08	F5, A3	*c*	110	− 4	**+1·35**
25	β Pers	4·9	+40 46	2·3–3·5	B8		117	−14	**−0·08**

No.	Star	R.A. 1950·0	Dec. 1950·0	Vis. Mag.	Spectrum H.D.	Remarks	Galactic Long.	Galactic Lat.	Gradient *G*
		h m	° ′				°	°	
26	ι Pers	3 5·4	+49 25	4·17	Go		112	− 6	+1·10
27	α Pers	20·7	49 41	1·90	F5	*c*	114	− 5	+0·88
28	ξ Taur	24·5	9 34	3·75	B8		142	−36	−0·02
29	δ Pers	39·4	47 38	3·10	B5		118	− 5	−0·16
30	o Pers	41·2	32 8	3·94	B1		128	−17	+0·33
31	η Taur	44·5	23 57	2·96	B5p	*e*	135	−22	−0·02
32	ζ Pers	51·0	31 44	2·91	B1	*c*	130	−16	+0·35
33	ϵ Pers	54·5	39 52	2·96	B1		125	− 9	−0·23
34	ξ Pers	55·7	35 39	4·05	Oe5		128	−12	+0·21
35	λ Taur	57·9	12 21	3·3–4·2	B3		146	−28	0·00
36	48 Pers	4 5·0	47 35	4·03	B3	*e*	121	− 2	+0·06
37	μ Pers	11·2	48 17	4·28	Go	*c*	122	− 1	+1·68
38	μ Taur	12·8	8 46	4·32	B3		152	−27	+0·12
39	53 Pers	17·9	46 23	4·89	B3		124	− 1	+0·01
40	κ^1 Taur	22·4	22 11	4·36	A3		142	−17	+0·27
41	τ Taur	39·2	22 52	4·33	B5		144	−13	−0·16
42	α Caml	49·1	66 16	4·38	Bo	*e*	111	+15	+0·17
43	7 Caml	53·3	53 40	4·44	A2		122	+ 8	0·00
44	ϵ Auri	58·4	43 45	3·3–4·1	F5p	*c*	130	+ 3	+1·20
45	β Caml	59·0	60 22	4·22	Gop	*c*	117	+12	+1·49
46	η Auri	5 3·0	41 10	3·28	B3		133	+ 2	−0·22
47	α Auri	13·0	45 57	0·21	Go		130	+ 6	+1·54
48	γ Orio	22·4	6 18	1·70	B2		165	−14	−0·24
49	β Taur	23·1	28 34	1·78	B8		146	− 2	−0·14
50	114 Taur	24·6	21 54	4·83	B3		151	− 6	−0·07
51	32 Orio	28·1	5 55	4·32	B3		166	−14	−0·11
52	χ Auri	29·5	+32 9	4·88	B1	*c*	144	+ 1	+0·83
53	δ Orio	29·5	− 0 20	2·48	Bo		171	−16	−0·19
54	ϕ^1 Orio	32·1	+ 9 27	4·53	Bo		164	−11	−0·06
55	λ Orio	32·4	+ 9 54	3·49	Oe5		163	−10	−0·10
56	ι Orio	33·0	− 5 56	2·87	Oe5		177	−18	−0·17
57	ϵ Orio	33·7	− 1 14	1·75	Bo		173	−16	−0·12
58	ζ Taur	34·7	+21 7	3·00	B3p	*e*	153	− 4	−0·17
59	139 Taur	54·9	25 57	4·90	B2		152	+ 2	+0·11
60	β Auri	55·9	44 57	2·07	A0p		135	+12	+0·06
61	θ Auri	56·3	37 13	2·71	A0p		142	+ 8	−0·05
62	χ^2 Orio	6 0·9	20 8	4·71	B2p	*e c*	157	+ 1	+0·77
63	ν Gemi	26·0	20 15	4·06	B5	*e*	160	+ 6	−0·13
64	13 Mono	30·2	7 22	4·50	A0p	*c*	172	+ 1	+0·28
65	γ Gemi	34·8	16 27	1·93	A0		164	+ 6	+0·15
66	15 Mono	38·2	9 57	4·68	Oe5		171	+ 4	−0·19
67	ξ Gemi	42·5	12 57	3·40	F5		168	+ 6	+0·87
68	θ Gemi	49·5	34 1	3·64	A2		150	+16	+0·23
69	15 Lync	53·0	58 29	4·54	Go		125	+25	+1·50
70	ζ Gemi	7 1·1	+20 39	3·7–4·3	Gop	*c*	163	+13	+1·39

No.	Star	R.A. 1950·0	Dec. 1950·0	Vis. Mag.	Spectrum H.D.	Remarks	Galactic Long.	Galactic Lat.	Gradient *G*
		h m	° ′				°	°	
71	δ Gemi	7 17·1	+22 5	3·51	Fo		164	+17	+0·74
72	21 Lync	22·9	49 19	4·45	Ao		136	+27	+0·05
73	β C Min	24·4	8 23	3·09	B8	*e*	177	+13	**−0·02**
74	α Gemi	31·4	32 0	1·58	Ao		155	+24	**+0·11**
75	α C Min	36·7	5 21	0·48	F5		181	+14	+0·83
76	*o* U Maj	8 26·1	60 53	3·47	Go		123	+36	+1·55
77	η Hyda	40·6	3 35	4·32	B3		191	+28	−0·16
78	ε Hyda	44·1	6 36	3·48	F8		189	+30	+1·39
79	ι U Maj	55·8	48 14	3·12	A5		139	+42	+0·43
80	α Canc	55·8	12 3	4·27	A3		185	+35	+0·34
81	κ U Maj	9 0·2	47 21	3·68	Ao		140	+43	+0·06
82	38 Lync	15·7	37 1	3·82	A2		154	+46	+0·20
83	θ U Maj	29·5	51 54	3·26	F8*p*		132	+47	+0·97
84	ε Leon	43·0	24 0	3·12	Go*p*	*c*	175	+50	+1·55
85	21 L Min	10 4·5	35 29	4·47	A5		157	+56	+0·37
86	η Leon	4·6	17 0	3·58	Ao*p*	*c*	188	+52	+0·17
87	α Leon	5·7	12 13	1·34	B8		195	+50	**−0·08**
88	λ U Maj	14·1	43 10	3·52	A2		142	+56	+0·10
89	ρ Leon	30·2	9 34	3·85	Bo*p*	*c*	204	+54	+0·04
90	37 L Min	35·9	32 14	4·77	Go	*c*	163	+62	+1·48
91	54 Leon	52·9	25 1	4·32	Ao		180	+65	+0·10
92	β U Maj	58·8	56 39	2·44	Ao		115	+56	**−0·02**
93	δ Leon	11 11·5	20 48	2·58	A3		194	+68	**+0·29**
94	θ Leon	11·6	15 42	3·41	Ao		205	+66	+0·07
95	ξ U Maj	15·5	31 49	3·86	Go		162	+70	+1·21
96	β Leon	46·5	14 51	2·23	A2		222	+72	+0·17
97	γ U Maj	51·2	53 58	2·54	Ao		106	+62	−0·05
98	δ U Maj	12 13·0	57 19	3·44	A2		98	+60	+0·14
99	β C Ven	31·4	41 38	4·32	Go		98	+76	+1·12
100	κ Drac	31·4	70 4	3·88	B5*p*	*e*	91	+48	−0·08
101	23 Coma	32·4	22 54	4·78	Ao		245	+85	+0·06
102	ρ Virg	39·4	10 31	4·95	Ao		267	+73	+0·20
103	ε U Maj	51·8	56 14	1·68	Ao*p*		87	+61	−0·05
104	α^2 C Ven	53·7	38 35	2·90	Ao*p*		79	+79	**−0·06**
105	78 U Maj	58·6	56 38	4·89	Fo		86	+61	+0·78
106	β Coma	13 9·5	28 8	4·32	Go		2	+84	+1·10
107	20 C Ven	15·3	40 50	4·66	Fo		64	+75	+0·53
108	ζ^1 U Maj	21·9	55 11	2·40	A2*p*		78	+62	+0·16
109	80 U Maj	23·2	55 15	4·02	A5		78	+62	+0·35
110	24 C Ven	32·4	49 16	4·63	A3		69	+66	+0·26
111	B.D. 37° 2426	32·6	37 26	4·96	Fo		45	+76	+0·87
112	25 C Ven	35·2	36 33	4·92	Fo		42	+76	+0·50
113	η U Maj	45·6	49 34	1·91	B3		65	+65	**−0·23**
114	η Boöt	52·3	18 39	2·80	Go		333	+72	+1·07
115	α Drac	14 3·0	+64 37	3·64	Ao*p*		76	+51	0·00

No.	Star	R.A. 1950·0	Dec. 1950·0	Vis. Mag.	Spectrum H.D.	Remarks	Galactic Long.	Galactic Lat.	Gradient *G*
		h m	° ′				°	°	
116	B.D. 3° 2867	14 9·7	+ 2 39	4·90	A0*p*		313	+57	−0·01
117	κ^2 Boöt	11·7	52 1	4·60	A5		62	+61	+0·44
118	ι Boöt	14·4	51 36	4·78	A5		61	+61	+0·45
119	λ Boöt	14·5	46 19	4·26	A0		51	+64	+0·16
120	B.D. 9° 2882	20·9	8 40	4·87	A0		325	+60	+0·21
121	γ Boöt	30·1	38 32	3·00	F0		32	+65	+0·40
122	σ Boöt	32·5	29 58	4·48	F0		11	+66	+0·78
123	π Boöt	38·4	16 38	4·54	A0		344	+61	+0·06
124	ζ Boöt	38·8	13 57	3·86	A2		339	+60	+0·07
125	γ U Min	15 20·8	72 1	3·14	A2		75	+41	+0·19
126	ν^2 Boöt	30·0	41 4	4·98	A2		32	+53	+0·18
127	θ Cor B	30·9	31 32	4·17	B5	*e*	17	+53	−0·11
128	α Cor B	32·6	26 53	2·31	A0		9	+53	+0·01
129	ζ Cor B	37·5	36 48	4·69	B8		25	+52	−0·08
130	ι Serp	39·3	19 50	4·49	A2		359	+49	+0·15
131	γ Cor B	40·6	26 27	3·93	A0		9	+51	+0·06
132	β Serp	43·9	15 35	3·74	A2		354	+46	+0·25
133	ζ U Min	45·8	77 57	4·34	A2		79	+36	+0·14
134	χ Herc	50·9	42 35	4·61	G0		34	+49	+1·15
135	Gr 2296	56·6	54 53	4·96	A5		52	+46	+0·60
136	ι Cor B	59·4	29 59	4·91	A0		15	+47	+0·03
137	π Serp	16 0·1	22 57	4·82	A2		5	+45	+0·17
138	υ Herc	1·2	46 10	4·64	B9		39	+47	−0·09
139	ϕ Herc	7·2	45 4	4·26	B9*p*		37	+46	−0·02
140	τ Herc	18·2	46 26	3·91	B5		39	+44	−0·19
141	ω Herc	23·1	14 9	4·53	A0*p*		357	+37	+0·10
142	15 Drac	28·1	68 53	4·98	B8*p*		68	+38	+0·05
143	σ Herc	32·5	42 32	4·25	A0		34	+42	+0·04
144	ζ Herc	39·4	31 42	3·00	G0		20	+39	+1·17
145	52 Herc	47·8	46 4	4·86	A2*p*		38	+39	+0·24
146	ι Ophi	51·6	10 15	4·29	B8		356	+29	−0·06
147	ϵ Herc	58·4	31 0	3·92	A0		20	+35	−0·01
148	60 Herc	17 3·1	12 48	4·91	A3		1	+28	+0·30
149	ζ Drac	8·6	65 47	3·22	B5		63	+35	−0·09
150	δ Herc	13·0	24 54	3·16	A2		14	+30	+0·20
151	68 Herc	15·5	33 9	4·8–5·3	B3		24	+32	−0·18
152	69 Herc	15·9	37 21	4·80	A2		28	+33	+0·10
153	ρ Herc	22·0	37 11	4·14	A0		28	+32	+0·04
154	β Drac	29·3	52 20	2·99	G0	*c*	46	+33	+1·61
155	ν^1 Drac	31·2	55 13	4·98	A5		50	+32	+0·52
156	ν^2 Drac	31·3	55 12	4·95	A5		50	+32	+0·65
157	α Ophi	32·6	12 36	2·14	A5		4	+21	+0·30
158	ι Herc	38·1	46 2	3·79	B3		39	+30	−0·24
159	δ U Min	48·3	86 37	4·44	A0		86	+29	+0·08
160	66 Ophi	57·8	+ 4 22	4·81	B3	*e*	358	+12	−0·03

No.	Star	R.A. 1950·0	Dec. 1950·0	Vis. Mag.	Spectrum H.D.	Remarks	Galactic Long.	Galactic Lat.	Gradient *G*
		h m	° ′				°	°	
161	67 Ophi	17 58·1	+ 2 56	3·92	B5*p*	*c*	357	+11	+0·25
162	72 Ophi	18 5·0	9 33	3·73	A3		4	+13	+0·25
163	*o* Herc	5·6	28 45	3·83	Ao		23	+20	+0·07
164	102 Herc	6·6	20 48	4·32	B3		15	+17	−0·10
165	ϕ Drac	21·5	71 19	4·24	Ao*p*		69	+28	+0·05
166	39 Drac	23·2	58 46	4·85	A2		55	+26	+0·22
167	α Lyra	35·2	38 44	0·14	Ao		35	+18	0·00
168	ϵ^1 Lyra	42·7	39 37	4·68	A3		36	+17	+0·37
169	ϵ^2 Lyra	42·7	39 34	4·50	A5		36	+17	+0·49
170	ζ^1 Lyra	43·0	37 33	4·29	A3		35	+16	+0·42
171	ζ^2 Lyra	43·1	37 32	5·87	A3		35	+16	+0·69
172	111 Herc	44·8	18 7	4·37	A3		16	+ 8	+0·22
173	β Lyra	48·2	33 18	3·4–4·1	B8, B2	*e c*	30	+14	+0·24
174	γ Lyra	57·1	32 37	3·30	Ao*p*		31	+12	0·00
175	ζ Aqil	19 3·1	13 47	3·02	Ao		15	+ 2	+0·04
176	η Lyra	12·1	39 4	4·46	B3		38	+12	−0·12
177	1 Vulp	14·1	21 18	4·60	B5		22	+ 3	+0·04
178	π Drac	20·4	65 37	4·63	A2		64	+21	+0·14
179	3 Vulp	20·8	26 10	4·92	B5		27	+ 4	−0·03
180	2 Cygn	22·2	29 31	4·86	B3		31	+ 6	−0·04
181	δ Aqil	19 23·0	3 1	3·44	Fo		7	− 7	+0·63
182	ι Cygn	28·4	51 37	3·94	A2		51	+15	+0·34
183	8 Cygn	29·9	34 21	4·85	B3		36	+ 7	−0·11
184	9 Vulp	32·4	19 40	4·88	B8		23	− 1	−0·05
185	α Sgte	37·9	17 54	4·37	Go	*c*	22	− 4	+1·39
186	δ Cygn	43·4	45 0	2·97	Ao		46	+10	−0·05
187	ζ Sgte	46·8	19 1	4·95	A2		24	− 5	+0·18
188	α Aqil	48·3	8 44	0·89	A5		15	−11	+0·42
189	12 Vulp	48·9	22 29	4·91	B3	*e*	27	− 3	−0·16
190	13 Vulp	51·3	23 57	4·50	Ao		29	− 3	0·00
191	22 Cygn	54·1	38 21	4·87	B3		41	+ 4	−0·01
192	ψ Cygn	54·3	52 18	4·80	A3		54	+12	+0·30
193	15 Vulp	59·0	27 37	4·74	A5		33	− 2	+0·37
194	28 Cygn	20 7·6	36 41	4·82	B2*p*	*e*	41	+ 1	−0·06
195	κ Ceph	10·6	77 34	4·40	B9		77	+22	+0·05
196	30 Cygn	11·7	46 40	4·96	A2		50	+ 6	+0·27
197	ρ Aqil	12·0	15 3	4·96	Ao		24	−12	+0·14
198	33 Cygn	12·2	56 25	4·32	A3		58	+12	+0·29
199	29 Cygn	12·7	36 39	4·98	Ao		42	0	+0·36
200	B.D. 25° 4165	13·1	25 26	4·82	B3		33	− 6	−0·27
201	P Cygn	15·9	37 53	4·88	B1*p*	*e*	43	+ 1	+1·03
202	γ Cygn	20·4	40 6	2·32	F8*p*	*c*	46	+ 1	+1·11
203	ω^1 Cygn	28·5	48 47	4·89	B3		54	+ 5	0·00
204	θ Ceph	28·7	62 50	4·28	A5		65	+14	+0·37
205	ϵ Dlph	30·8	+11 8	3·98	B5		23	−18	−0·12

No.	Star	R.A. 1950·0	Dec. 1950·0	Vis. Mag.	Spectrum H.D.	Remarks	Galactic Long.	Galactic Lat.	Gradient *G*
		h m	° ′				°	°	
206	ζ Dlph	20 33·0	+14 30	4·69	A2		26	−16	+0·22
207	β Dlph	35·2	14 25	3·72	F5		27	−17	+0·82
208	α Dlph	37·3	15 44	3·86	B8		28	−16	+0·02
209	α Cygn	39·7	45 6	1·33	A2*p*	*e c*	52	+ 1	Var.
210	Pi 20^h 332	44·1	57 24	4·63	Go		62	+ 9	+0·97
211	λ Cygn	45·5	36 18	4·47	B5	*e*	46	− 5	−0·01
212	55 Cygn	47·2	45 56	4·89	B2	*e c*	54	+ 1	+0·99
213	57 Cygn	51·5	44 12	4·68	B3		53	− 1	−0·11
214	ν Cygn	55·3	40 58	4·04	Ao		51	− 3	+0·10
215	59 Cygn	58·1	47 19	4·86	Bo*p*	*e*	56	+ 1	−0·07
216	τ Cygn	21 12·8	37 50	3·82	Fo		51	− 8	+0·85
217	σ Cygn	15·4	39 11	4·28	Ao*p*	*c*	52	− 8	+0·42
218	υ Cygn	15·9	34 41	4·42	B3*p*	*e*	49	−11	−0·05
219	68 Cygn	16·6	43 44	5·06	Oe5		56	− 4	+0·24
220	α Ceph	17·4	62 22	2·60	A5		68	+ 9	**+0·43**
221	β Ceph	28·0	70 20	3·32	B1		75	+14	−0·30
222	9 Ceph	36·6	61 51	4·87	B2*p*	*c*	70	+ 7	+0·70
223	π^1 Cygn	40·3	50 58	4·78	B3		63	− 2	−0·06
224	ν Ceph	44·0	60 53	4·46	A2*p*	*c*	70	+ 6	+1·29
225	π^2 Cygn	44·9	49 5	4·26	B3		63	− 4	+0·04
226	14 Pegs	47·6	29 56	5·00	Ao		51	−19	−0·01
227	ξ Ceph	22 2·3	64 23	4·40	A3, G		74	+ 7	+0·62
228	θ Pegs	7·7	5 57	3·70	A2		36	−40	+0·17
229	ε Ceph	13·2	56 48	4·23	Fo		70	0	+0·61
230	2 Lacr	19·0	46 17	4·66	B5		66	− 9	−0·08
231	32 Pegs	19·0	28 5	4·88	B8		55	−24	+0·08
232	31 Pegs	19·1	11 57	4·93	B3*p*	*e*	44	−37	−0·03
233	4 Lacr	22·5	49 13	4·64	B8*p*	*c*	68	− 7	+0·30
234	δ Ceph	27·3	58 10	(3·6–4·3)	F2–G2		73	+ 1	Var.
235	6 Lacr	28·3	42 52	4·54	B3		65	−13	−0·02
236	α Lacr	29·2	50 1	3·85	Ao		69	− 7	+0·04
237	9 Lacr	35·3	51 17	4·83	A5		71	− 6	+0·48
238	10 Lacr	37·0	38 47	4·91	Oe5		65	−17	−0·15
239	ζ Pegs	39·0	10 34	3·61	B8		48	−41	−0·11
240	ο Pegs	39·4	29 3	4·85	Ao		60	−26	+0·05
241	η Pegs	40·7	29 58	3·10	Go		61	−25	+1·53
242	ο Andr	59·6	42 3	3·63	B5, A2	*c*	70	−16	−0·05
243	α Pegs	23 2·3	14 56	2·57	Ao		57	−41	−0·03
244	1 Cass	4·5	59 9	4·93	B1	*c*	78	− 1	+0·12
245	τ Pegs	18·2	23 28	4·65	A5		66	−35	+0·33
246	υ Pegs	22·9	23 8	4·57	Go		67	−36	+1·09
247	B.D. 57° 2748	27·7	58 16	4·89	B3		80	− 3	−0·10
248	ι Andr	35·7	42 59	4·28	B8		77	−18	−0·01
249	κ Andr	37·9	44 3	4·33	Ao		78	−17	−0·05
250	σ Cass	56·5	+55 29	4·93	B2		83	− 6	+0·08

THE RELATIONS BETWEEN COLOUR INDICES AND GRADIENTS

R. d'E. Atkinson, A. Hunter and E. G. Martin

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In addition to the "ordinary" colour index, defined as the difference between photographic and visual magnitude, there are now available a number of lists of special colour indices, obtained either photographically or photo-electrically, but in any case differing from each other in their sensitivity ranges. Many of these lists are of high internal accuracy, but none makes any claim to be fundamental, and most of them are limited in scope. Indeed there is no really extensive catalogue even of uniform and reliable photographic magnitudes that might be used as a comprehensive, though empirical, basis for comparisons; many of the magnitudes given in H.D. (*H.A.* 90–99) are simply assumed from the spectra and visual magnitudes. The overlap between the various lists of actually determined colour indices permits some degree of inter-comparison, and some such comparisons have been made, but it is difficult to discuss these on any theoretical basis.

The completion of the Greenwich "colour temperature" programme has provided a list of 250 "gradients" of considerable accuracy. This also is only a small number of stars, but the selection is a useful one; nearly all stars of types O, B and A north of the equator and down to $4^{m}\cdot5$ are included, as well as a considerable proportion of those between $4^{m}\cdot5$ and $5^{m}\cdot0$ and a small selection of F and G types. For stars of these early and medium types, gradients can claim a real theoretical importance; in the present case the wave-lengths of measurement do not extend far into the violet, and of course avoid all appreciable lines; and a number of tests failed to establish any systematic (or even any large random) departure from a black-body distribution in the continuum for the region considered. The probable errors (about 0·02 for the average "programme" star and 0·013 for "standard" stars), though a little larger than those of the best colour indices, are certainly not large enough to offset this theoretical advantage, and it is thus of interest to compare colour indices with Greenwich gradients.

The publication of the Greenwich volume containing the full details of the work has unfortunately had to be postponed as a result of the war. The actual values have, however, been published*, and the following outline of the experimental procedure will suffice for the present purpose.

Slitless spectra of a "programme star" and a "standard star" are obtained at equal altitudes in rapid succession; the plate is calibrated, and (visual) photometer measurements of the continuum at two regions, in the red and blue respectively, give the quantity $\Delta m_r - \Delta m_b$, where Δm

* *M.N.*, **100**, 189, 1940.

is the magnitude difference of the two stars (being reckoned positive when the comparison is in the sense *bright* minus *faint*), and the suffixes denote the colours. When this difference is divided by $\Delta(1/\lambda) = (1/\lambda)_b - (1/\lambda)_r$ and multiplied by $\frac{2}{5}\log_e 10 = 0{\cdot}921$ it is called the "difference of gradient" or "relative gradient" of the two stars; it is almost independent of the wave-lengths of measurement over a wide range, if the spectrum is that of a black body. By a proper choice of zero, so far arbitrary, for the whole system, the individual gradients can be converted to "absolute gradients" ϕ, where, as is well known*,

$$\phi = \frac{c_2}{T}\left(1 - e^{\frac{-c_2}{\lambda_m T}}\right)^{-1}$$

in which T is the "colour temperature" of the star and λ_m is an approximate mean wave-length for the spectral range employed. However, this aspect of the matter is irrelevant for our purpose. The absolute value of the zero for the Greenwich system is less well defined than the relative gradients, so that the temperatures are also comparatively uncertain. The following discussion does not depend on temperatures at all, but only on the assumption that, if the effect of line absorption could be entirely eliminated, stellar spectra would be nearly those of black bodies; it can thus be made entirely on the basis of (relative) gradients, G. Small departures from a black-body curve for the continuum would also not invalidate it.

It is clear that, algebraically speaking, these gradients are defined just as colour indices are, *i.e.* they are essentially the difference between a red magnitude and a blue magnitude for the star. They contain, of course, a zero for the whole system, and the factor $\dfrac{0{\cdot}921}{\Delta(1/\lambda)}$; but apart from the additive constant (which is irrelevant) and this scale factor, they are formally identical with colour indices. Physically they are by no means identical, and straight-line correlations between colour indices and gradients are not in general to be expected; the actual property of the star that is measured is definitely different in the two cases.

In the Greenwich procedure for determining gradients, the spectra are measured at about six to eight selected wave-lengths in the blue, and again in the red, and the values are averaged to give a mean blue magnitude-difference Δm (*i.e.* difference between two stars) at a mean blue wave-length, and similarly in the red. (The differences of magnitude are in theory almost exactly linear in $1/\lambda$, and the harmonic mean of the wave-lengths is thus the one used.) Even in F and G stars it is possible to pick the individual wave-lengths so that all lines of strength greater than 2 (Rowland) are reasonably well avoided. For the most part the wave-lengths in each region lie fairly close together, and even within this short range the values of Δm show signs of the expected run in $1/\lambda$. The final magnitude-difference adopted, either for the red or for the blue, represents then quite clearly the true difference between the two continua at the mean

* *Observations of Colour Temperatures of Stars, 1926–1932*, p. 2 (H.M. Stationery Office, 1932).

red or blue wave-length involved; the influence of absorption-lines is eliminated, and no question arises of a sensitivity varying over the range of measurement, as in the case with work through filters.

[The photographic reductions are performed in the first instance as though the exposures on the two stars had been equal, which would normally be the case only if their mean magnitudes had been approximately equal. Actually of course there may be a considerable difference in magnitude and so also in exposure time; the effect of this is, however, roughly the same at both colours, so that it cancels out on differencing, and in the above description it has been omitted for simplicity. In the reductions themselves it is carefully corrected for by an ingenious method due to Professor W. M. H. Greaves. The correction is negligible for bright stars; for faint yellow ones it sometimes exceeded 0·07 in G, in which case the result was half-weighted in forming the final value; in general it was less than 0·04.]

If colour indices also could eliminate the effect of line absorption, and had uniquely defined effective wave-lengths, then, provided they were confined to wave-lengths within which the same black-body curve applies as fits the gradient work, they would correlate linearly with gradients, the relation being

$$\frac{dC}{dG} = \frac{\Delta(1/\lambda)}{0{\cdot}921}, \qquad (1)$$

where C is the colour index and $\Delta(1/\lambda)$ refers now to the two effective wave-lengths *used in determining* C. The arbitrary constant introduced by integrating this depends on the zeros chosen for C and for G; in point of fact, it is fairly usual to define C so that $C=0$ for the average of some selection or other of A0 stars, and this is true of the present *relative* gradients G also; but the selections employed vary from one case to another. The discussion can ignore this point and deal in slopes only.

It is found, when colour indices are plotted in terms of G, that the above relation is not normally satisfied. Qualitatively, of course, the reason is clear; the determinations of magnitude that are made for colour-index work take the average intensity (weighted in accordance with the particular sensitivity-curve involved) over a considerable range of wave-lengths which frequently includes a number of extremely strong absorption lines. The correlation shows in fact marked curvature in some cases, and when the so-called "reddened B stars" are included it may cease to be even single-valued.

These reddened stars are of special interest in the present discussion. They may be loosely defined as stars whose gradients are appreciably larger than would be expected for their spectral class; the excess may in fact be very considerable. Some of them have been known for fifteen years and more, while others are recent discoveries. (Among the stars too faint for Greenwich gradient work there are some that appear to be even more strongly affected than any Greenwich stars.) Nearly all are of early spectral type (B3 or earlier), but very marked reddening is also possible at later types; ν Cephei, cA2, has a gradient of $+1{\cdot}29$ (equal to that of an average

Go star) where about +0·2 would be expected. They are all unquestionably very luminous, so that even near the type Ao their total line absorption is relatively fairly small; their continua also seem to fit black-body curves (though of a temperature very low for their type) at least as well as those of ordinary stars and perhaps better. Thus although the cause of the reddening itself is still imperfectly known, they are particularly easy to discuss from the point of view of colour index.

Before doing so, however, we will summarise the evidence for the statement that their continuous spectra, in which the energy-distributions are certainly very different from those suggested by their ionisation temperatures, are nevertheless approximately those of black bodies. None of these stars seems as yet to have been directly compared with a black body in the Greenwich wave-length range. They have, however, been compared with normal stars, including both stars of the same type and stars of the same colour. Struve, Keenan and Hynek * examined 55 Cygni and 13 Cephei, and found (roughly speaking) that the differences in magnitude taken against an early B star were linear in $1/\lambda$ over a wide range from λ 3500, that they were effectively zero throughout when taken against a normal F star, and that by comparison with late B and early A stars the continua were less depressed in the violet and ultra-violet, *i.e.* where normal stars are known to fall below the black-body intensity. Their measurements are only preliminary, corrections for variation of photographic γ with λ, and for difference of exposure time, being omitted. At Greenwich certain reddened stars were examined from the point of view of possible departures from a black-body distribution within the Greenwich wave-length range only; such a departure would be inferred if the value of Δm measured at some intermediate wave-length did not interpolate linearly (in $1/\lambda$) between the values for the red and blue. Accordingly a group of suitable wave-lengths was selected in the green, and measurements were made at these points in addition to the usual ones. The following table shows the resulting mean residuals at the (harmonic) mean wave-length λ 5308, in magnitudes, for suitable plates of five reddened stars taken against Greenwich standard stars:—

TABLE A

	γ Cass	χ^2 Orio	P Cygn	55 Cygn	ν Ceph
	+0·04	−0·02	+0·03	−0·02	+0·02
	−0·08	+0·05	+0·04	+0·07	+0·01
	+0·03	+0·10	+0·03	+0·05	−0·03
	+0·10	+0·18	+0·01	−0·10	−0·09
			+0·08	−0·08	
Mean	+0·02	+0·08	+0·04	−0·02	−0·02

Owing to the fact that none of these plates was exposed for the purpose of this check, and that the low sensitivity of the emulsion in the green has therefore resulted in under-exposure, the measures might be expected to be less certain than usual; in fact this effect is hardly noticeable, and so far as

* *Ap. J.*, 79, 1, 1934.

they go the results certainly fail to indicate any significant deviation for abnormally reddened stars.*

Accordingly, if we now consider all the early B stars, whether reddened or not, we have a group showing a wide range of colours, but all approximately fitting black-body curves, and all with small (sometimes negative) total line-absorption. We can fairly confidently predict that for these stars the relation between gradient and colour index will in fact be the simple "theoretical" one given by (1).

Fig. 1 shows the relation between gradient and the colour index determined photo-electrically by Elvey † with the Yerkes 40-inch refractor. Reddened stars are indicated by name ; doubtful or weak reddening is indicated by brackets. (Stars denoted only by numbers or by spectral types are not reddened.) The short isolated straight line is drawn at the slope given by (1), Elvey's values (λ 3850 and λ 5100) for his effective wave-lengths being adopted to derive $\Delta(1/\lambda)$. All normal stars earlier than B3 are indicated by spectral type only. It is at once evident that the early-type stars, normal and reddened together, do satisfy an approximately linear relation between C and G, but the slope is not quite that of (1). The exact slope is not very well defined, but it is certain that much of the apparent scatter is systematic, as it correlates directly with spectral type ; the dotted line divides stars of type B2 and earlier from those of type B3 and later, and among the reddened stars only one, the exceptional P Cygni, falls in the wrong group. Probably the best estimate of the slope can be obtained from the Bo and B1 stars only, but although it is larger than for all the named stars taken together, it is still not more than about 0·56, whereas using Elvey's figures the theoretical value is 0·691. If the arguments presented here are justified, it is difficult to explain the discrepancy except on the assumption that Elvey's effective wave-lengths are not quite correct. There appear to be three possible factors that might in fact have produced some error in them.

(*a*) The ultra-violet side of his short-wave range was defined only by an assumed transmission of the 40-inch lens. This appears ‡ to have been estimated from that for two plane-parallel plates of crown and flint glass of normal properties, and it might be in error if the glass actually was less transparent. In any case, if the flint is appreciably less transparent than the crown in the ultra-violet, as seems likely, the effective aperture would be cut down at the shortest wave-lengths, since for these the thick flint lens would more than compensate for the thin crown lens at the edge of the combination.

(*b*) An alternative factor that may have cut down the aperture in the ultra-violet is introduced by the use of a diaphragm at the focus of the 40-inch lens. The lens being a visual achromatic, the longitudinal chromatic aberration at λ 4000 would presumably be of the order of 1/200 of the focal length,§ or

* Compare also earlier work by Trumpler, *P.A.S.P.*, **42**, 267, 1930.

† *Ap. J.*, **74**, 298, 1931.

‡ This procedure was at least employed later, by Stebbins and Huffer, *Publ. Washb. Obs.*, **15**, 218, 1934.

§ See, for example, Conrady, *Applied Optics and Optical Design* (Pt. 1), diagram on p. 167 (Oxford, 1929). Curves given by the firm of Carl Zeiss in an advertisement "Astro. 34" confirm this figure qualitatively. Neither authority actually deals with such short wave-lengths, however.

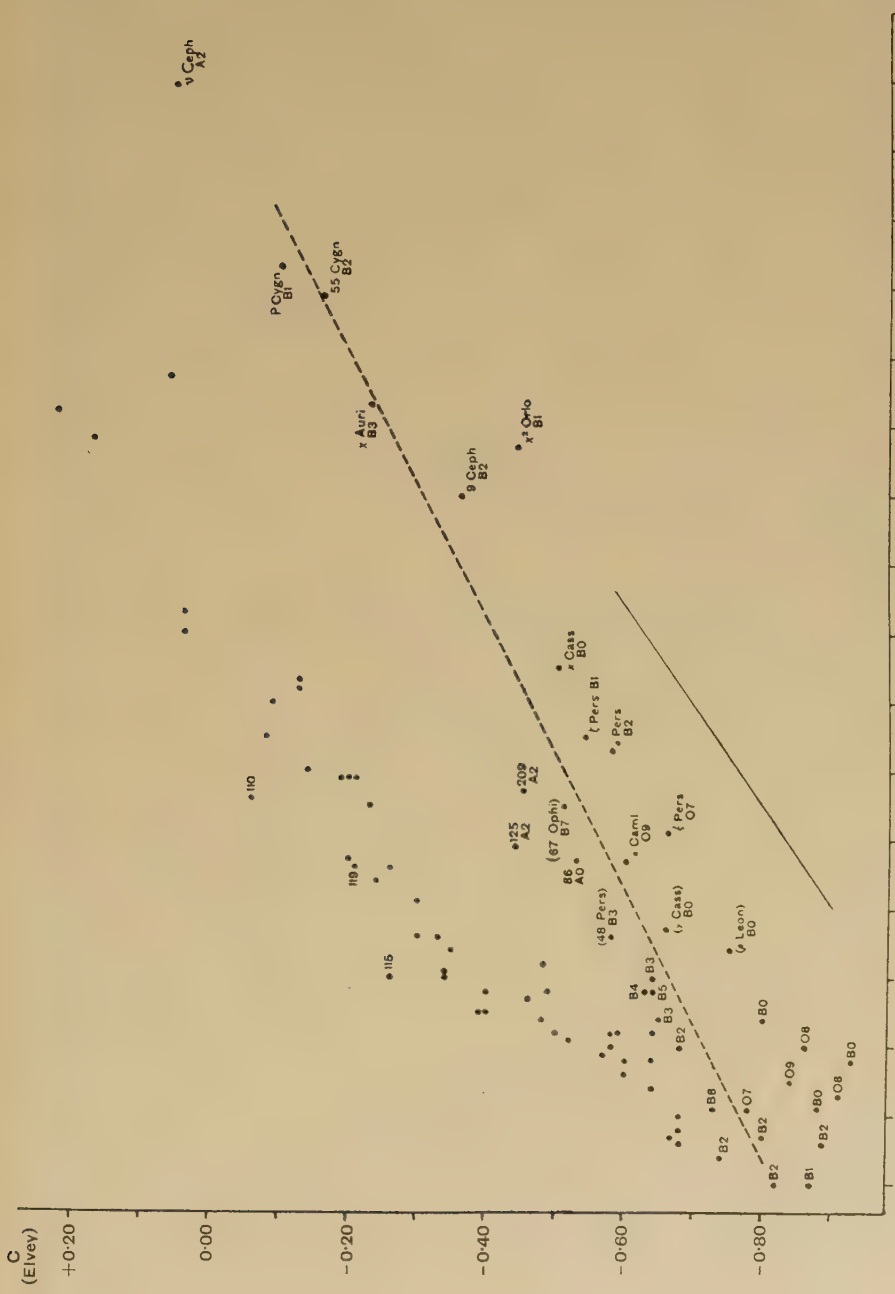
C (Elvey)
+0·20
0·00
-0·20
-0·40
-0·60
-0·80
0·00
+0·50
+1·00
Q
ν Ceph A2
P Cygn B1
55 Cygn B2
ι Aurl B3
9 Ceph B2
χ² Orio B1
χ Cass B0
ζ Pers B1
Pers B2
209 A2
110
125 A2
(67 Ophi) B7
Caml O9
ζ Pers O7
86 A0
(48 Pers) B3
(ι Cass) B0
(ν Leon) B0
119
115
B4
B5
B3
B2
B3
B0
B8
O7
B2
B2
B2
B1
O9
O8
B2
O8
B0
B0
O8
B0

FIG. 1.

say 10 cm. An iris diaphragm was used at the (visual) focus *, giving apertures from 3·6 to 34·4 mm. ; if the smallest was used, the effective aperture at λ 4000 was cut down geometrically to 0·036, or $f/28$. The natural aperture of the lens being $f/19$, the loss of ultra-violet light (neglecting factor (*a*) above) is about 55 per cent., increasing rapidly with diminishing wave-length. It is not stated what aperture was in fact used, or whether the diaphragm setting was even varied ; the correlation shown in fig. 1 suggests, however, that it cannot have been varied much unless the aperture was really limited by factor (*a*).

(*c*) Elvey determined the transmission of his filters "roughly" by "photographing the sky spectrum with and without filter on a calibrated plate." The effective sensitivity depends of course on the product of the filter transparency and the cell sensitivity, and nothing is said as to how the latter was determined, or whether it was assumed. It is perhaps therefore possible that all determinations involved were rather rough, or that the sensitivity curve for the cell was (incorrectly) assumed to be (for example) similar to that of the Washburn photometer. Stebbins states † that "blue" and "yellow" filters in conjunction with the cell gave it effective wave-lengths in the neighbourhood of λ 4000 and λ 5000 ; the theoretical slope calculated from these figures is 0·543, which is smaller than is required by fig. 1.

All things considered then, it appears possible that the agreement with the theory developed here is satisfactory for these stars. If it should be found that Elvey's wave-lengths were not appreciably in error, it would be necessary to look elsewhere for an explanation of the slope in fig. 1, probably to residual departures from the black-body curve taken in conjunction with the fact that Elvey's work was done in the green and violet while the Greenwich work was done in the red and blue.

In discussing the normal stars it is desirable to be quite clear what the effect of absorption lines may be expected to be, since the conditions to be satisfied if the relation (1) is to hold are not in fact as strict as might appear at first sight. It is not, for example, necessary that the total line absorption in the "violet" range should be equal to that in the "green" range, still less, of course, that these absorptions should both be zero. The relation will hold for any group of stars for which the *difference* of these two absorptions is constant ; the absorptions must clearly be expressed in magnitudes down from the respective black-body curves *that fit the gradient determinations*. Thus a moderate amount of violet line absorption, such as might be found in a cA star, could perhaps be balanced by the absorption at $H\beta$ sufficiently nearly to account for the positions of 67 Ophiuchi (cB7), ν Cephei (cA2), η Leonis (cA0, No. 86 ‡) and α Cygni (cA2, No. 209 ‡) in fig. 1, whether or not such stars are reddened (η Leonis and α Cygni are not). It is of course possible in principle that line absorption in one part of the violet range should be balanced by, *e.g.*, fluorescent continuous

* *Ap. J.*, 74, 291, 1931. † *Ap. J.*, 74, 296, 1931.

‡ Here and elsewhere in this paper, numbers are the serial numbers of stars used in *Relative Gradients of 250 Stars*, *M.N.*, **100**, pp. 190–195.

emission in another part of the same range; but the only likely continuum, that of the Balmer series, is probably almost entirely outside the sensitivity range of the instruments in Elvey's work. For all practical purposes the relation will be satisfied if, and only if, the above condition holds.

"Normal" stars near type Ao have, of course, very strong hydrogen line absorption in the violet range, which is bound to increase the colour index above the value for a reddened B star of the same gradient. Thus with advancing type among the normal B stars the colour index shows at first a rapid increase, compared with the gradient, as a consequence of the rapidly growing Balmer lines. At somewhat later types, when the Balmer lines begin to fade, H and K produce the same general effect of holding the violet depressed.

If the increase in H and K just balances the decrease in the hydrogen line intensity, as weighted by the violet sensitivity curve of the photometer (or if any lack of balance can be made up by the decrease of $H\beta$, etc. in the green range), the C–G slope will now once more be given by (1) in spite of the very large total line absorption. This appears to be roughly true for Elvey's results from about $G=0$ (B8 stars) onwards, and on the whole it would seem reasonable to expect it, though a somewhat greater effect of H and K would not have been surprising; no G stars, however, are available for comparison. Thus the correlation, both for normal and for reddened stars, is in its general features what one might expect; but it is manifestly not suitable for very simple algebraic formulation.

The question which stars should be called "reddened" will sometimes be answered quite differently if colour indices are made the criterion instead of gradients. The early A stars 24 Can Ven (110), α Draconis (115) and λ Bootis (119), for example, are among the reddest of their spectral types on the basis of Elvey's work, but they are of quite normal gradient. In fig. 1 they fall definitely above the mean curve for "normal" stars; there seems no reason to doubt that the high colour index is simply due (as Elvey already suggested) to heavy Balmer line absorption in the violet range.

The importance of this factor has been noted by Elvey, Williams * and others; it is brought out particularly clearly in fig. 2, in which the ordinates are $G-C/0{\cdot}691$ and the abscissæ are the spectroscopic absolute magnitudes given by Öpik and Olmsted.† All stars common to Elvey's list and the Greenwich one, of H.D. types B8–A2 both inclusive, are shown, except υ Herc, which is not in *H.O.C.* 380. (For α Cygni, G has been found to be variable, but the value $G=0{\cdot}27$ has been used, as the mean obtained near the date of determination of C.) The difference between gradient and colour index (corrected for scale) is likely to be particularly significant for line absorption, because it is independent of any reddening factor that leaves the energy curve of the continuum still that of a black (or grey) body, whether this agent is stellar or interstellar.

There are a number of other bright stars of these types that are unfortunately not in Elvey's list, but even with the material available a clear

* *Ap. J.*, 79, 295, 1934. † *H.O.C.* 380, 1932.

correlation is evident. This is, of course, to be expected, since the criterion for absolute magnitude in *H.O.C.* **380** was total hydrogen line absorption for $H\gamma$ and $H\delta$, with practically the same calibration curve for types B8–A2. None the less it has an independent value. For the difference between gradient and colour index determines this quantity in a way that cannot be disturbed by axial rotation nor (for the Greenwich wave-length range) by the difficulty often experienced in estimating where the continuum really

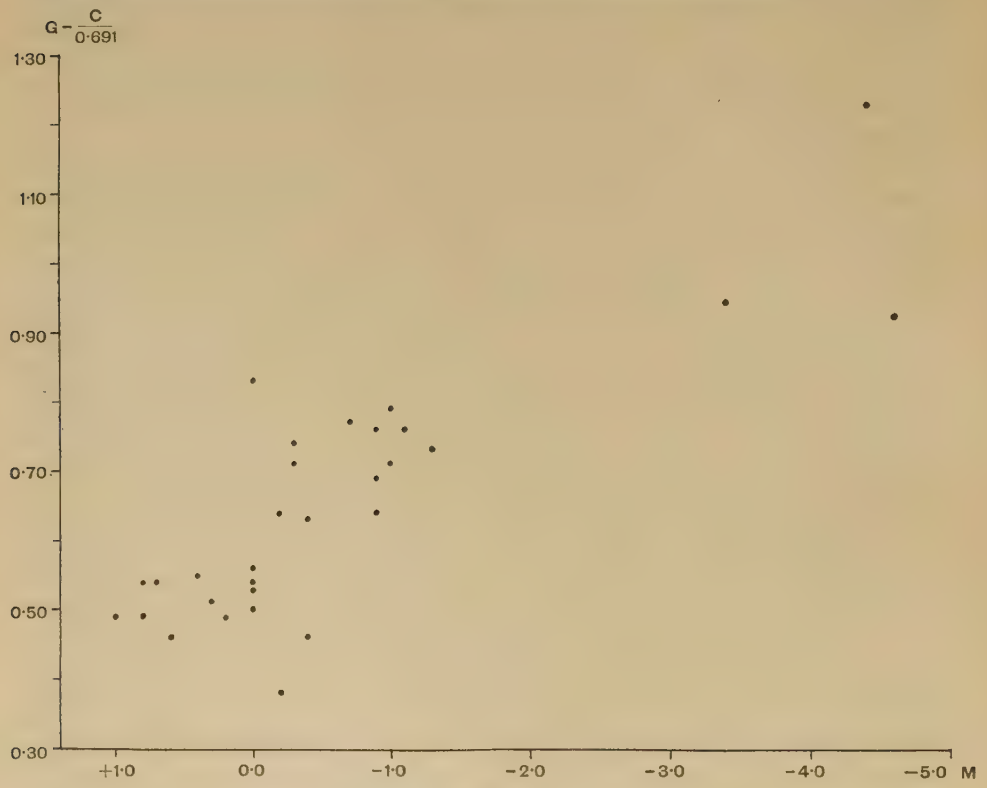

FIG. 2.

is when the wings of strong lines overlap. (On the other hand, of course, it can be affected by H and K.)

From fig. 2 we see that the "dwarf reddening" amounts to 0·7 on the gradient scale, for a difference of $5\frac{1}{2}$ magnitudes in total light. This is equivalent to half a magnitude more loss of light in Elvey's violet range than in his green one, or 37 per cent. of the total available light in this range, when dwarfs are compared with super-giants of the same spectral type. Much the same figure was obtained by Williams.* The difference in principle between gradients and colour indices (which has sometimes been overlooked) is evident from fig. 1, but its significance is brought out best by this correlation. Considering the number and strength of the Balmer lines in Elvey's violet region and the extent to which their wings

* *Ap. J.*, 79, 297, 1934.

are known to overlap in dwarfs, it does not appear that the loss of light is greater than could be accounted for by them alone.

It may be noted that a similar distinction between normal and reddened stars is found even when the ultra-violet *gradients* of Barbier, Chalonge and their collaborators * are plotted against Greenwich gradients. By analogy with colour indices this certainly suggests once more that there is no quite undisturbed continuum below, say, λ 4200 in main sequence A stars, but only in super-giants, so that "gradients" taken at short wave-lengths must be interpreted with great caution. This point, however, requires further study.

The apparent gap between the normal and reddened stars in fig. 1 may now seem surprising, since there are quite a number of only moderately reddened stars. The gap is, however, probably not altogether real; of the eight Greenwich stars of types from B5 to A5 whose absolute magnitudes (*H.O.C.* 380) lie between $-1{\cdot}5$ and -3, only two are in Elvey's list, and both of these (η Leonis (No. 86) and 67 Ophiuchi) actually lie on the edge of the gap. For early B stars, however, a clear separation from normal A, F and G stars is certainly to be expected. An early B star, whether it is much reddened or little, will never have strong overlapping hydrogen or Ca^+ lines, and any reddening of the black-body type will simply move the corresponding point along the "theoretical" slope, which is practically the slope for the reddened stars.

Elvey's results have been discussed in some detail, because the sensitivity ranges happen to be such that the features of the comparison are particularly well marked. In other cases both the separation between normal and reddened stars and the curvature of the correlation for normal ones are less striking. We may, however, consider a few cases.

Stebbins and Huffer † used a photometer essentially the same as that of Elvey, attached to the 15·6-inch Washburn (visual) refractor. The filters were much less dense, owing to the desire to measure fainter stars with a smaller telescope, and the two sensitivity ranges now overlapped appreciably. The effective wave-lengths are given (for equal energies) as λ 4260 and λ 4770; but with sensitivity ranges as wide as these were, the real effective wave-lengths will (as the authors pointed out later) ‡ depend appreciably on the colour of the star; for an early B star they become λ 4170 and λ 4710. The shorter effective wave-length may here again be too short (in either case); the factors (*a*) and (*b*) above (p. 200) may apparently have been operative in this work also. Their list of stars includes all northern stars of H.D. types B5 and earlier to about $7^{m}{\cdot}5$, but only a small number later than B5 were taken, so that the number available for comparison with gradients is not greater than before. The comparison is shown in fig. 3; the "theoretical" slope is based on the effective wave-lengths λ 4260 and λ 4770.

The line for the reddened stars has once more a slope smaller than the "theoretical" one, but if the shorter effective wave-length were really λ 4360 the agreement would be good. Examination of the sensitivity curve given

* *Ann. d'Astroph.*, **1**, 293 and 402, 1938.
† *Publ. Washb. Obs.*, **15**, 217, 1934.
‡ *Ap. J.*, **90**, 213, 1939.

by the authors suggests that if factor (*a*) or (*b*) was actually operative, the wave-length might well have been as long as this. The correlation for the normal stars is effectively linear; this indicates that the line absorption in the violet range increased steadily with advancing spectral type, which is probably reasonable, since the higher hydrogen lines cannot have had as much effect here as H and K.

Appreciably earlier, Bottlinger *, and also Becker †, determined photo-electric colour indices for a large number of stars, with high internal accuracy.

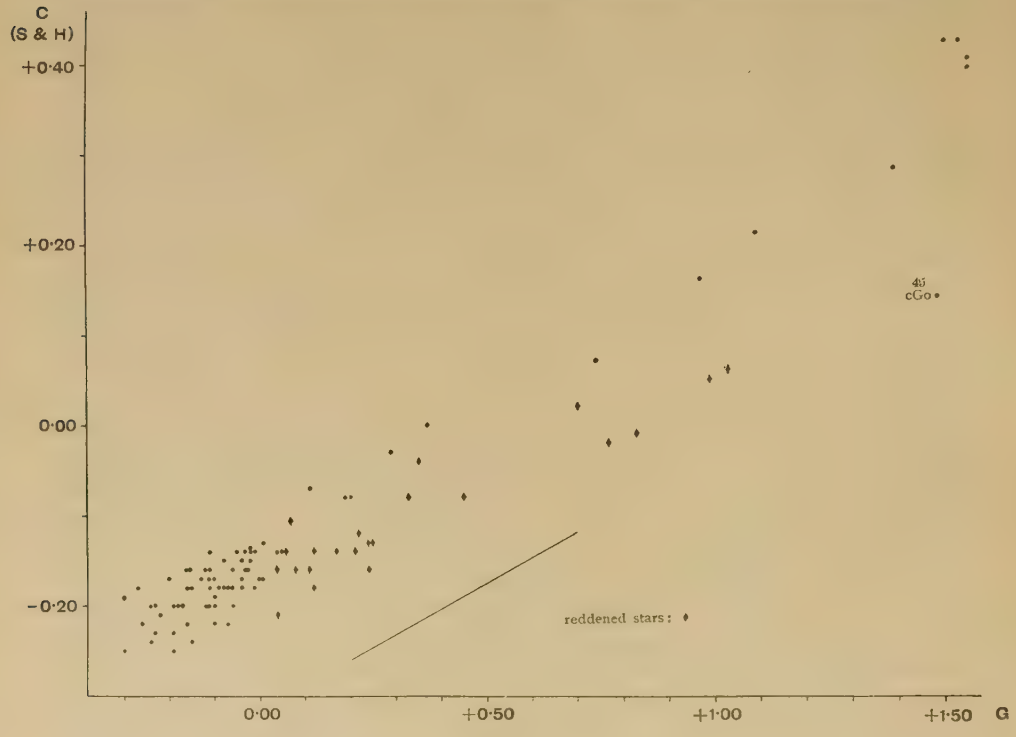

FIG. 3.

Bottlinger's results correlate linearly with those of Stebbins and Huffer (*loc. cit.*, p. 241) and of Elvey (*loc. cit.*, p. 302), but as his effective wave-lengths are not stated it is difficult to draw any definite conclusion. The graph (fig. 4) is intermediate in type between figs. 1 and 3, which presumably explains how the results correlate linearly with both. There is no separation of reddened B stars from other stars in either correlation. The normal yellow stars are much better represented in this comparison with *G* than in the others, and the figure suggests that the effect of H and K increases greatly for these stars.

Hall ‡ used a Cs – O cell with and without an infra-red colour filter. The resulting correlation with gradients is almost entirely single-valued, the reddened B stars showing scarcely any tendency to "anomalous" behaviour,

* *Veröff. Berlin-Bab.*, **3**, 4, 1923. † *Veröff. Berlin-Bab.*, **10**, 3, 1932.
‡ *Ap. J.*, **79**, 145, 1934.

but it is by no means linear. No definite effective wave-lengths are given, and the sensitivity curve of the cell shows that the interpretation would be complex. The behaviour of the reddened stars is, however, not surprising.

When we come to discuss photographic magnitudes, and the colour indices determined from them with the aid of visual magnitudes, we are faced again with the difficulty of deciding what is the sensitivity range of the measurements. In the case of reflectors it would be nearly that of the photographic emulsion itself, or of the eye, as the case may be ; most photographic magnitudes, however, have been determined with refractors, and

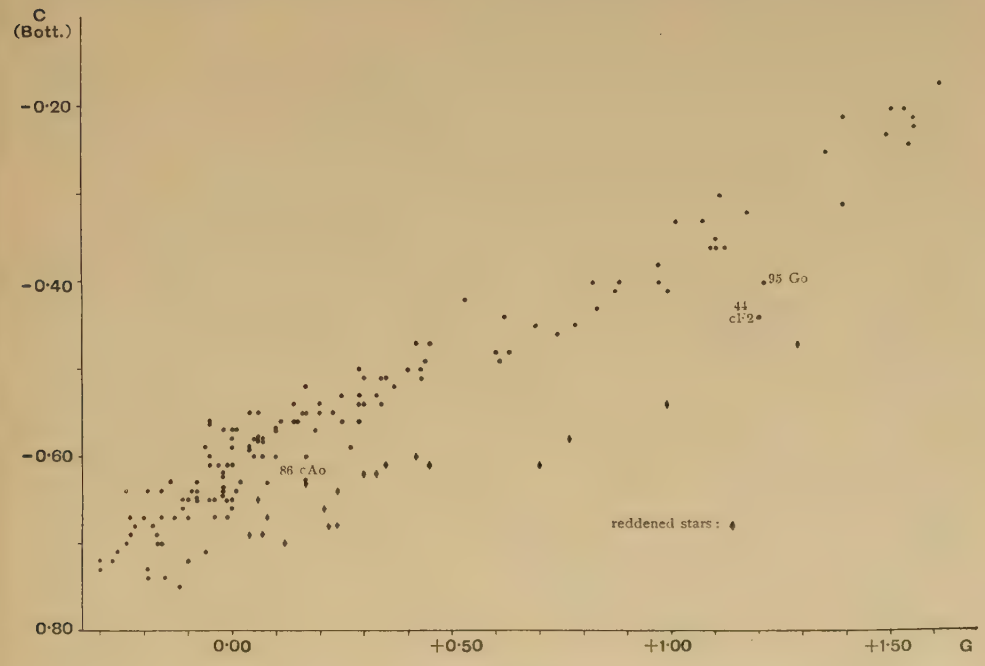

FIG. 4.

most at least of the HR visual magnitudes were also determined with visual achromatics of the ordinary type.

While the sensitivity curve for the eye with a visual achromatic is fairly well known, it may be pointed out that calculations of it have generally ignored the influence of the "secondary spectrum" of the objective. The minimum focal length for such telescopes is usually at about λ 5550 (rather longer with large instruments), and while the range of wave-lengths that is in useful focus varies both with the relative aperture and with the magnification, it is never as large as with a reflector. (Very large refractors, in fact, are almost colour-blind.) It seems probable that an effective wave-length of say λ 5300, calculated without considering this effect, is really too short.

Photographic magnitudes may be determined either focally or extra-focally ; in the former case the range of effective wave-lengths will be smaller than in the latter, for the same reason. In both cases the exact range is difficult to estimate theoretically.

Fig. 5 shows the relation between gradient and the colour index obtained by differencing King's photographic magnitudes (*H.A.*, **76**, 117) and HR visual magnitudes (*H.A.*, **50**). King's magnitudes were obtained extra-focally, for the most part with a visual refractor fitted with a photographic correcting lens. It is probable that this and the other lenses used had minimum focal lengths in the neighbourhood of λ 4300; since the plates were taken extra-focally, the wave-lengths longer and shorter than this figure would form extra-focal patches of somewhat smaller size, *i.e.* greater

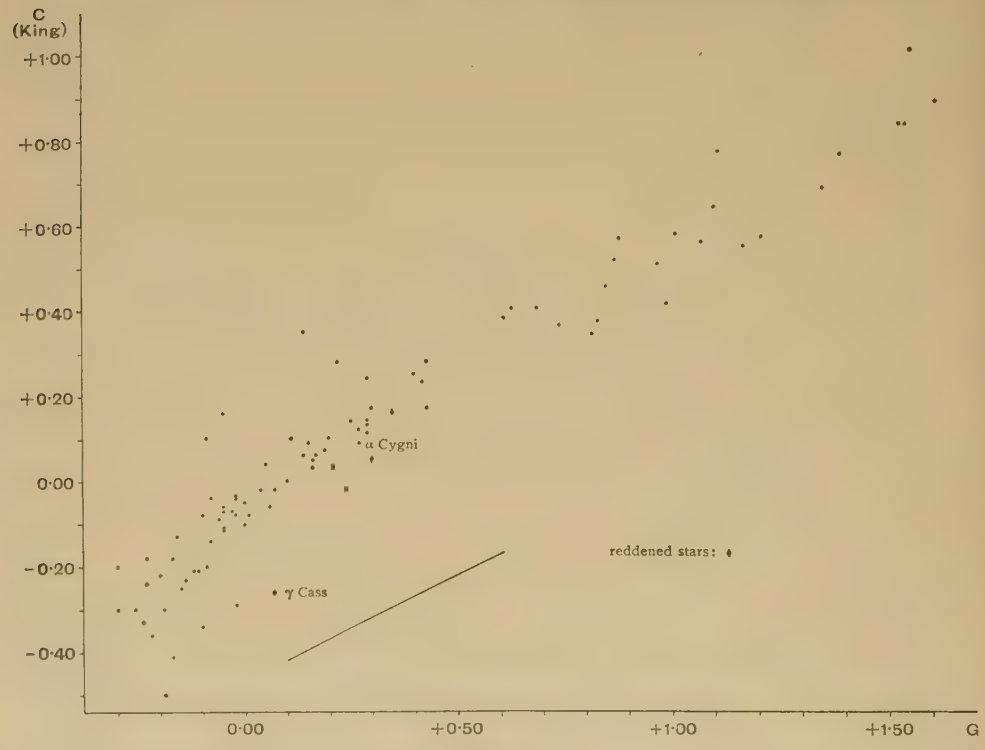

Fig. 5.

intensity, than those at the minimum, but it appears unlikely that this effect can have been large. The total loss of light, as compared with a pinhole photometer, was given as $0^m.66$, and most of this would presumably be at short wave-lengths.

Discussion of fig. 5 is complicated by the unfortunate accident that it includes no strongly reddened stars, and no c stars earlier than F4 except the moderately reddened stars shown (and also α Cygni, which is variable). Neglecting this star, and γ Cass for the same reason (the difference of epoch introduces an uncertainty), there is evidently very little indeed to go on in deciding what the true slope would be to compare with (1). Presumably it would be rather less than the mean slope for the diagram; this is 0·63 from a least-squares solution that uses all the stars, and (1) would give this value if the effective wave-lengths were, for example, λ 4200 for photographic

magnitudes and λ 5550 for visual. The true effective wave-lengths are therefore closer together than these figures. The straight line indicates the slope calculated from the values λ 4260 and λ 5290*; this slope appears a little small, *i.e.* it departs from the observed values in the opposite sense to that of figs. 1 and 3.

The sensitivity range in King's work must probably have been limited on the ultra-violet side, since the moderately reddened stars are mainly of quite early type, and should stand off from normal stars of the same gradient if the effect of hydrogen line absorption were appreciable for the latter.

Even without allowing for any small systematic effect of this sort, the correlation in fig. 5 is very good. The residuals in the least-squares solution (which assumed a linear relationship) give a probable error of $\pm$ 0·050 in colour index, and this could have been slightly reduced by assuming a curved relation and excluding the reddened stars. Thus, even allowing for the natural uncertainty in a probable error derived from only about 90 residuals, this figure is unlikely to be an under-estimate. The mean probable error of the visual magnitudes in HR, obtained from the residuals there quoted for 90 of the stars in fig. 5, is $\pm$ 0·037; the corresponding quantity for the photographic magnitudes (44 stars only; the remainder are assumed to have probable errors $\sqrt{2}$ times as large) is $\pm$ 0·031, making $\pm$ 0·048 for the colour indices. The balance is evidently without significance; formally, however, it could be made up by a probable error of $\pm$ 0·013 in colour index, which represents, if the slope be taken as 0·63, a probable error of $\pm$ 0·02 in gradient; this is exactly the Greenwich average estimate. There is thus apparently nothing that can be attributed to systematic effects; in point of fact it is clear that some of the probable errors have been over-estimated, since small systematic effects are evident in fig. 5. The probable errors for both the visual and the photographic magnitudes are really very uncertain, and all that can be said is that the correlation appears better than would be expected.

Nearly all the Greenwich stars are included in Hertzsprung's list of 658 photographic magnitudes †, so that a comparison between colour indices, obtained from these, and gradients is of interest on account of the wealth of material. The comparison is given in fig. 6, in which the colour index is derived, as before, with the help of HR visual magnitudes. It is evident that though the scatter is larger in this case the actual relationship is well determined. It is definitely a little less linear than in fig. 5.

An interesting list of colour indices that may usefully be compared with gradients is that to be obtained by subtracting any consistent set of visual magnitudes from the photographic magnitudes of the original Draper Catalogue (*H.A.*, **27**). These magnitudes are of less accuracy, but they

* These values were taken from Russell, Dugan and Stewart. Brill (*Veröff. Berlin-Bab.*, VII, 5, 7, 1929) gives slightly different figures, but an appreciable dependence on spectral type is shown in all his work, and a constant figure is indeed theoretically wrong whenever the sensitivity range is wide. The actual effect on $\Delta(1/\lambda)$ is, however, small as a rule.

† *Bull. Astr. Inst. Neth.*, **1**, 201, 1923.

deserve perhaps more attention than they have received; they represent monochromatic *continuum* intensities measured in the spectra, with a wedge, at a wave-length a little longer than $H\gamma$; the photographic calibration was derived from the large number of A stars on each plate, all obtained with the same exposure time (objective-prism spectra), whose photographic magnitudes were taken to be equal to their visual ones on the average.

Fig. 7 shows colour indices obtained by differencing the photographic magnitudes of *H.A.*, **27**, and HR visual magnitudes. The crosses indicate

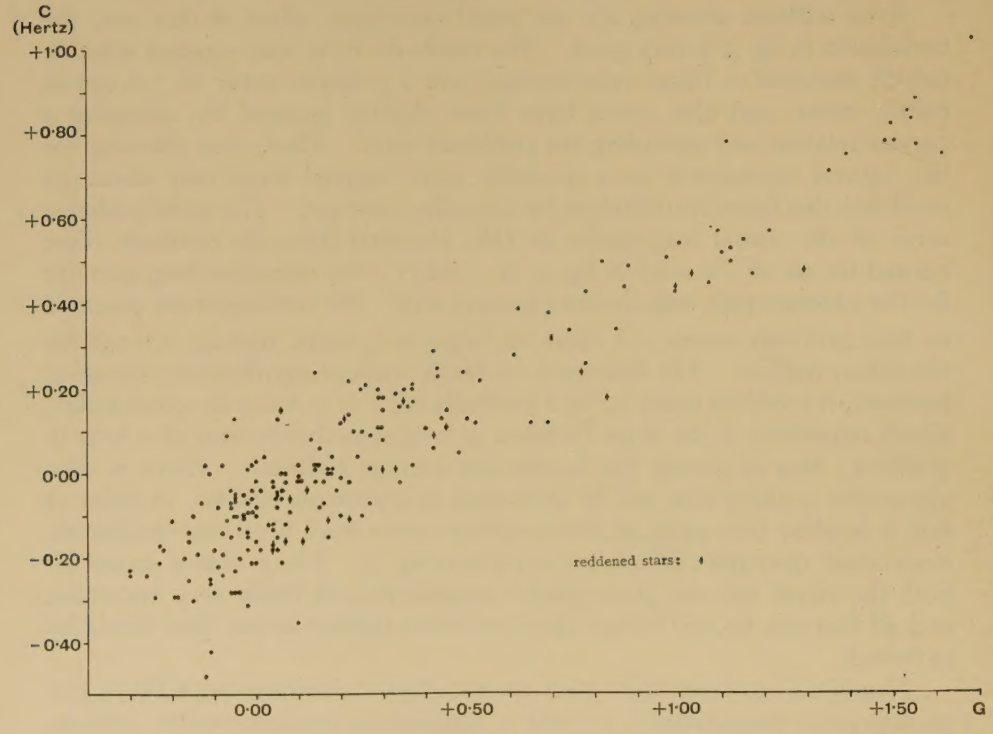

FIG. 6.

stars whose photographic magnitudes are fainter than 4·80, and the circles those between 4·20 and 4·80; for those brighter than 4·20 there is a serious upward scatter, and evidently for bright stars the photographic calibration was systematically at fault. The probable error obtained from the residuals from a curve drawn arbitrarily, omitting stars brighter than $4^m.8$ and also reddened stars, is about $0^m.08$; $0^m.044$ is due to HR (as judged by the residuals given for these stars), leaving a maximum (if the errors of the gradients are entirely ignored) of $0^m.068$ for the probable error of the photographic magnitudes. The mean slope is not very well determined, but corresponds approximately to effective wave-lengths of λ 4350 and λ 5500. As λ 4350 cannot be appreciably in error in this case, the value for HR is somewhat strengthened.

The reddened stars again show a slight systematic deviation, though it is

somewhat masked by the scatter. If it is real, it probably indicates that for normal stars the region measured was depressed by an unsuspected wing of $H\gamma$. Apart from this effect, and from the uncertainty in the effective wave-length of HR, these colour indices are nearly entitled to be called gradients, since the effect of absorption lines in the visual region is neither large nor seriously variable.

A comparison of the various diagrams makes it clear that comparatively

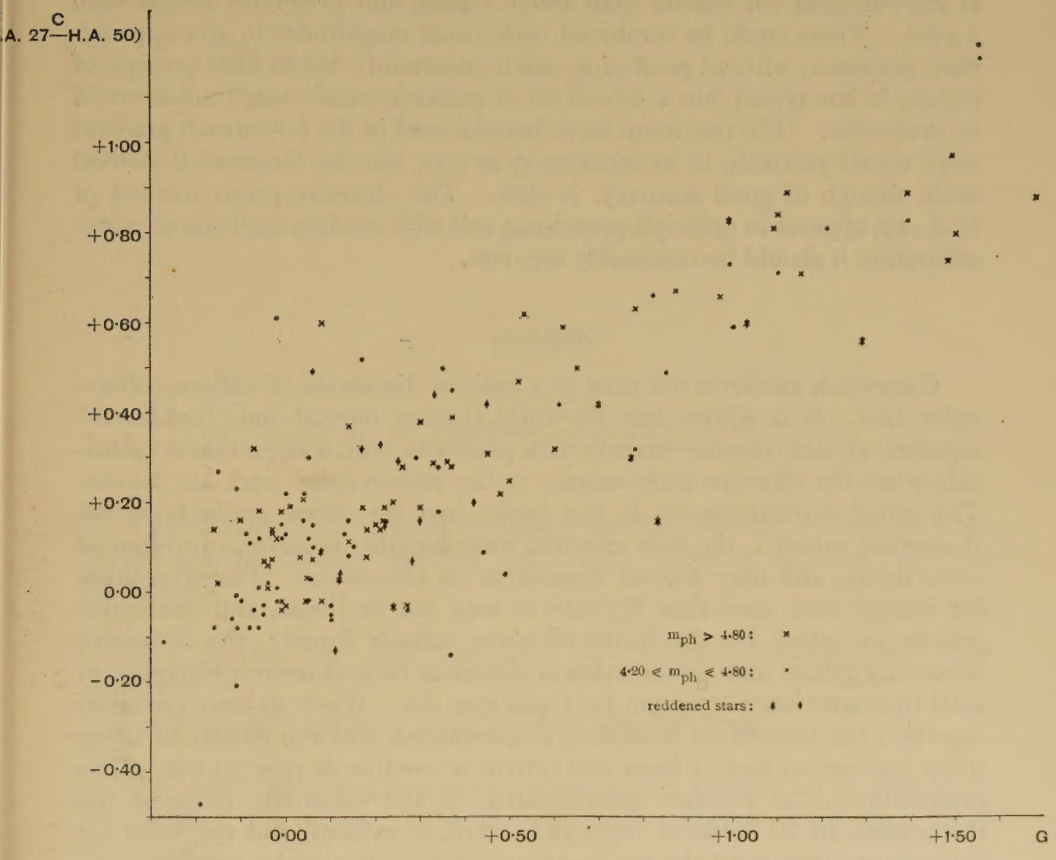

FIG. 7.

small changes in effective wave-lengths can sometimes produce appreciable systematic changes in the resulting colour indices. Further, the fact that the sensitivity range is often rather extensive deprives the conception of "effective wave-length" of much of its usefulness, more particularly when *part* of the range includes strong absorption lines for *some* of the spectral types studied and not for others. It is also evident that the existence of linear correlations may be deceptive; it would certainly be wrong, for example, to infer (constant) effective wave-lengths from the slope for the normal stars in fig. 3 (Stebbins and Huffer), and in addition the fact that both their results and Elvey's appear to correlate linearly with Bottlinger's has proved no guarantee that if the one correlated linearly with G the other

would also. The assumption of linear relationships on grounds of arithmetical convenience may clearly be unjustified.

It appears from this discussion that where magnitudes are determined over a finite range of wave-lengths it is desirable to have an accurate absolute determination of the true sensitivity throughout the range. From the point of view of theoretical application, however, "monochromatic" magnitudes appear even more desirable, and in order to be safe these should be taken at wave-lengths not shorter than about λ 4200, and preferably longer than λ 4500. These could be combined with visual magnitudes to give approximate gradients, without producing much uncertainty due to lines (except, of course, in late types), but a second set of monochromatic magnitudes would be preferable. The two mean wave-lengths used in the Greenwich gradient work would probably be as satisfactory as any, but the Greenwich method itself, though of good accuracy, is slow. The objective-prism method of *H.A.*, **27**, appears in principle promising, and with modern methods of plate-calibration it should be reasonably accurate.

Summary

Greenwich gradients are used as a basis of discussion of various colour-index lists. It is shown that the early B stars, normal and "reddened" together, should correlate linearly with gradients, with a slope that is calculable when the effective wave-lengths in the colour-index work are known. The actual correlations are in fact linear, and the slopes probably fit the theoretical values : the true effective wave-lengths, however, are often in some doubt, and may depend themselves on the colour. The correlation for normal stars later than B3 may or may not be linear, and qualitative reasons are given for the forms of curve actually found ; the difference between gradient and colour index is shown to be a measure principally of total hydrogen line absorption for types near A0. When all stars are taken together, the correlation is seldom single-valued, and the danger of using linear conversion factors from one system to another is pointed out. The desirability of an accurate determination of the sensitivity range of the instrument, in all kinds of magnitude work, is evident, and the need for monochromatic magnitudes for larger numbers of stars is also stressed.